SOP No. 29

Standard Operating Procedure for the Assignment of Uncertainty

1. Introduction

1.1. Purpose
Laboratories performing calibrations that meet ISO/IEC Guide 25, ISO/IEC 17025, or ANSI/NCSL Z 540-1-1994 must report uncertainties in conformance with the 1993 *ISO Guide to the Expression of Uncertainty in Measurement (*hereafter called the *GUM)*. This SOP provides instruction for the laboratory to meet this requirement.

1.2. Prerequisites

1.2.1. Calibration certificates with valid uncertainties must be available for all standards.

1.2.2. Statistical data regarding the calibration measurement process must be available; preferably from measurement control programs within the laboratory.

1.2.3. Knowledge of the technical basis for the measurement is critical for completeness in uncertainty evaluation. This can be obtained through brainstorming, experimentation, interlaboratory comparisons, cause and effect diagrams and the like. Flow charts at the end of this SOP show a number of common factors affecting measurements in the areas of mass, length, and volume.

2. Methodology

2.1. Scope, Precision, Accuracy

Each measurement made in a laboratory has a corresponding uncertainty assigned to the calibration value. The uncertainty is directly related to the measurement parameter (scope), range of the measurement, the equipment or measurement process being used (affecting precision), and the standards available with associated uncertainties.

2.2. Summary

This uncertainty analysis process follows the following steps:
1) the measurement process is clearly specified;
2) uncertainty sources are identified and characterized;
3) uncertainty sources are quantified;
4) uncertainty sources are converted to standard uncertainties;
5) the combined uncertainty is calculated;
6) the expanded uncertainty is calculated;
7) the expanded uncertainty is evaluated against appropriate tolerance or calibration limits; and
8) measurement results are reported.

Special methods for handling bias/errors are addressed as well.

3. The Process of Measurement Uncertainty Estimation

3.1. Step 1. Specify the process.

Clearly specify the measurement process in question, including the item being measured and the input quantities upon which it depends. This will usually require a quantitative expression related to the process. Where possible, you may reference an SOP or other method description along with the specific standards and measurement assurance process that is used to adequately complete this step.

3.2. Step 2. Identify and characterize uncertainty sources.

Identify all possible sources of uncertainty in a comprehensive list, characterizing them based on the evaluation method that will be used to quantify them (Type A, statistical methods or Type B, scientific judgement) and to categorize them based on their relatedness with something such as a flow chart (shown as Appendices), a cause and effect diagram, or an uncertainty budget table. Using the expression identified in 3.1 provides a good starting point. All of the parameters in this expression may have an uncertainty associated with them. When there are discrete steps in the measurement process, additional uncertainties may be associated with each.

What follows are the most common uncertainties associated with metrological measurements. Keep in mind that this list is exhaustive. Each item listed below is identified as a standard uncertainty u, when determined using Type B methods of evaluation and a standard uncertainty s, when determined with Type A methods of evaluation (statistical methods). Each standard uncertainty is further defined by an arbitrary subscript related to the source for ease in remembering that source.

3.2.1. Standard uncertainty from the measurement *process*, s_p, (Type A evaluation).

3.2.1.1. Standard deviation from a measurement assurance chart or control chart.

The value for s_p is obtained from the control chart limits and the current knowledge that the measurements are in a state of statistical control. This will have to be ascertained by measuring at least one check standard during the course of the current measurements.

3.2.1.2. Standard deviation from a series of measurements.

Measure a stable test object at least seven times, no two measurements of which may be made on the same day. Calculate the standard deviation in the conventional manner. The latter is the value of s_p, keeping in mind that it does not fully represent the measurement process under all typically encountered conditions.

Note: Repetitive measurements made on the same day estimate only the short-term standard deviation of the process.

3.2.2. Standard uncertainty for the *standards*, u_s (Type B evaluation).

3.2.2.1. When using standards calibrated by another laboratory.

The information for the standards comes from the calibration report, generally reported as an expanded uncertainty with its coverage factor (k). The expanded uncertainty is simply divided by k to obtain the combined uncertainty for the standard, u_c, which represents the u_s when used in your laboratory.

3.2.2.2. When using a standard calibrated in your laboratory (Type B evaluation).

If the standard was calibrated in your own laboratory, calculate the combined standard uncertainty, u_c, at k=1 and use that as the standard uncertainty for the standard, u_s.

3.2.2.3. When using more than one standard (Type B evaluation).

When more than one standard is used in a calibration, the standard uncertainty for each, u_{S1}, u_{S2}, u_{S3}, etc., is included in the RSS equation if the standards have had independent calibrations. When calibrations are performed at the same time, the standards may be dependent, so the standard uncertainties may be added ($u_{s1} + u_{s2}$) to determine a value to represent u_s. (This will be the case with two 1 kg standards that were calibrated at the same time using a weighing design and subsequently used together as standards (restraints) in a weighing design.)

3.2.3. Standard uncertainty due to *other* factors, u_o (Type B evaluation.)

These are factors related to the measurement equation, but distinct from the standard uncertainties associated with the process and the standards. These items are often much smaller in a well-controlled process than the standard uncertainties associated with the process and the standards. Examples are given in the flow charts.

The laboratory must carefully consider any other components of uncertainty that might contribute to the uncertainty of the measurement. In mass measurements, these might include effects from the magnitude of air buoyancy corrections as discussed in SOP 2, when these corrections are not made, or the uncertainty from the calculation of air density when buoyancy corrections are made (also discussed in SOP 2).

Additionally, the laboratory should include any other components that are not negligible, such as the uncertainty associated with the density of the standards, the density of the unknown test items, effects due to lack of thermal equilibrium during the test, or known and quantified effects due to magnetized mass standards. Each component that is considered should be included as an additional standard uncertainty u_{o1}, u_{o2}, u_{o3}, etc., and included in the RSS equation when data shows these factors to be significant. Note that components of uncertainty are not limited to those mentioned. Documentation of each component evaluated should be maintained to complete the documentation required by ISO/IEC 17025.

3.2.4. Standard uncertainty due to factors *unrelated* to the measurement process, u_u.

These are factors that may be related to characteristics of the items being testing or of the standard and are usually minimized in well-known and controlled measurement processes.

3.2.5. Special uncertainties from other sources (Type B evaluations). Includes bias or unidentified errors.

It is a general requirement of the GUM that corrections be applied for all recognized and significant systematic effects. Where a correction is applied based on a bias, an estimate of the associated uncertainty must then be included in the uncertainty analysis. Due to the various approaches present in the metrology system, several examples and possible approaches are presented in the section on calculating the combined or expanded uncertainties. At this stage, a determination must be made with regard to 1) identifying cause and 2) level of significance.

3.2.5.1. Identifying cause.

If the cause can be identified, it is usually corrected or applied to the measurement equation. In some cases, it is not possible to unarguably define the cause without exhaustive studies that provide little benefit. In those cases, the significance level must be evaluated.

3.2.5.2. Significance level.

When there is little to be gained from exhaustive studies on the measurement process to identify bias or potential errors, a test of significance may be conducted to determine alternative approaches for incorporating the bias into the uncertainty calculations.

In deciding whether a known bias can reasonably be included in the uncertainty, the following approach may be followed:

3.2.5.2.1. Estimate the combined uncertainty without considering the relevant bias.

3.2.5.2.2. Evaluate whether or not the bias is less than two times the combined uncertainty (i.e., $bias < 2\sqrt{u_s^2 + s_p^2 + u_o^2}$).

3.2.5.2.3. If the bias is less than twice the combined uncertainty, it may be included in the uncertainty using one of several approaches that must clearly be communicated in the report.

3.2.5.2.4. If the bias is larger than twice the combined uncertainty, the error must be investigated further prior to providing calibration data.

If the deviations show that a standard is out of control, it should not be used for calibration until corrective action has been taken and the value for the standard is verified as being within criteria limits.

$$criteria\ \text{limit}: \left| \bar{x}_{lab} - \text{x}_{\text{ref}} \right| < 2\,u_c$$

If these differences are smaller than the criteria limits, investigation and corrective action may be unrealistic. If the deviations are less than the surveillance limits shown above, and corrective action is not taken, the deviations may be included in the uncertainty statement following one of several options given in the following section. In ALL cases, the method used to incorporate bias must be clearly reported.

3.2.5.3. Adding the bias to the expanded uncertainty (e.g., used in PMAP software). In this case, the bias is simply added to the expanded

$$U + \text{bias} = (u_c * k) + \text{bias}$$

uncertainty and is reported as such.

3.2.5.4. Example when uncertainties for the laboratory data and the reference data are considered equivalent (e.g., laboratory data is compared to another laboratory's data).

In this case, a rectangular distribution is considered where the value might possibly be anywhere within the range shown for each laboratory data point. This method is referenced in section 4.6 of NIST Technical Note 1297. This approach may also be used in the case where a standard is predictably drifting with use over time. In this case, a mid-range value is chosen and u_d (uncertainty for *differences*) is calculated as follows:

$$u_d = \frac{\text{bias}}{2}\frac{1}{\sqrt{3}} \quad \text{or more simply: } 0.29\ \text{d, where d is the bias}$$

3.2.5.5. Example when uncertainties for the laboratory data are considered secondary to a reference value (e.g., the difference between the laboratory data and NIST data). This method was originally published in 1994.

In this case, a reference value is given precedence over the laboratory data and a mid-range value is not chosen. The extreme value is more probable. In this case, the following equation may be used:

$$u_d = \frac{\text{bias}}{\sqrt{3}} \quad \text{or more simply: } 0.577\ \text{d, where d is the bias}$$

3.3. Step 3. Quantify uncertainty estimates

All uncertainty estimates identified in the previous step must be quantified in units that represent the measured values. Type A methods of evaluation usually provide quantified estimates in the units of interest.

Type B methods of evaluation may be conducted with spreadsheets using the basic expression identified in the SOP or identified when the process was specified. Scenario testing can be done to determine the impact and quantify specific variables on the final measured quantity. The knowledge gained in this step often proves useful in identifying potential areas of improvement.

3.4. Step 4. Convert all factors to standard uncertainties

In those cases where the uncertainty factors were determined statistically (Type A methods), the standard deviation is used to represent the standard uncertainty. In other cases, estimates must be made to ensure that the quantified uncertainties represent "one-standard-deviation" values or a k=1 coverage level. The appropriate distribution factor must be used when converting values.

3.5. Step 5. Calculate the combined uncertainty

The combined standard uncertainty, u_c, includes the standard uncertainty reported for the standards used, u_s, the standard uncertainty of the measurement process, s_p, the standard uncertainty from other sources, u_o, which includes all other factors the laboratory considers significant, the standard uncertainty due to factors related to the measured item but unrelated to the measurement process, u_u, and finally, the standard uncertainty due to bias or differences, u_d, when u_d is included. The standard uncertainties are combined using the root-sum-of-the-squares (RSS) method as follows:

$$u_c = \sqrt{s_p^2 + u_s^2 + u_o^2 + u_u^2 + u_d^2}$$

Table 1. Symbol descriptions

Symbol	Description
U	Expanded uncertainty
u_c	combined standard uncertainty
s_p	standard uncertainty (standard deviation) of the "process"
u_s	standard uncertainty of the "standard"
u_o	standard uncertainty of "other factors"
u_u	standard uncertainty of factors "unrelated" to the measurement process
u_d	standard uncertainty of "differences" (may be treated in different ways)
k	coverage factor

3.6. Step 6. Calculate the expanded uncertainty

The combined standard uncertainty is then multiplied by a coverage factor, *k*, equal to 2 or 3, as chosen by the laboratory, to provide a level of confidence of approximately 95 % or 99 %, respectively. This procedure is based on the Central Limit Theorem and in most cases allows the use of 2 or 3 to provide an approximate 95 % or 99 % level of confidence. The equation used to determine the expanded uncertainty is as follows:

$$U = u_c * k$$

where $k = 2$ or $k = 3$.

3.7. Step 7. Evaluate the expanded uncertainty

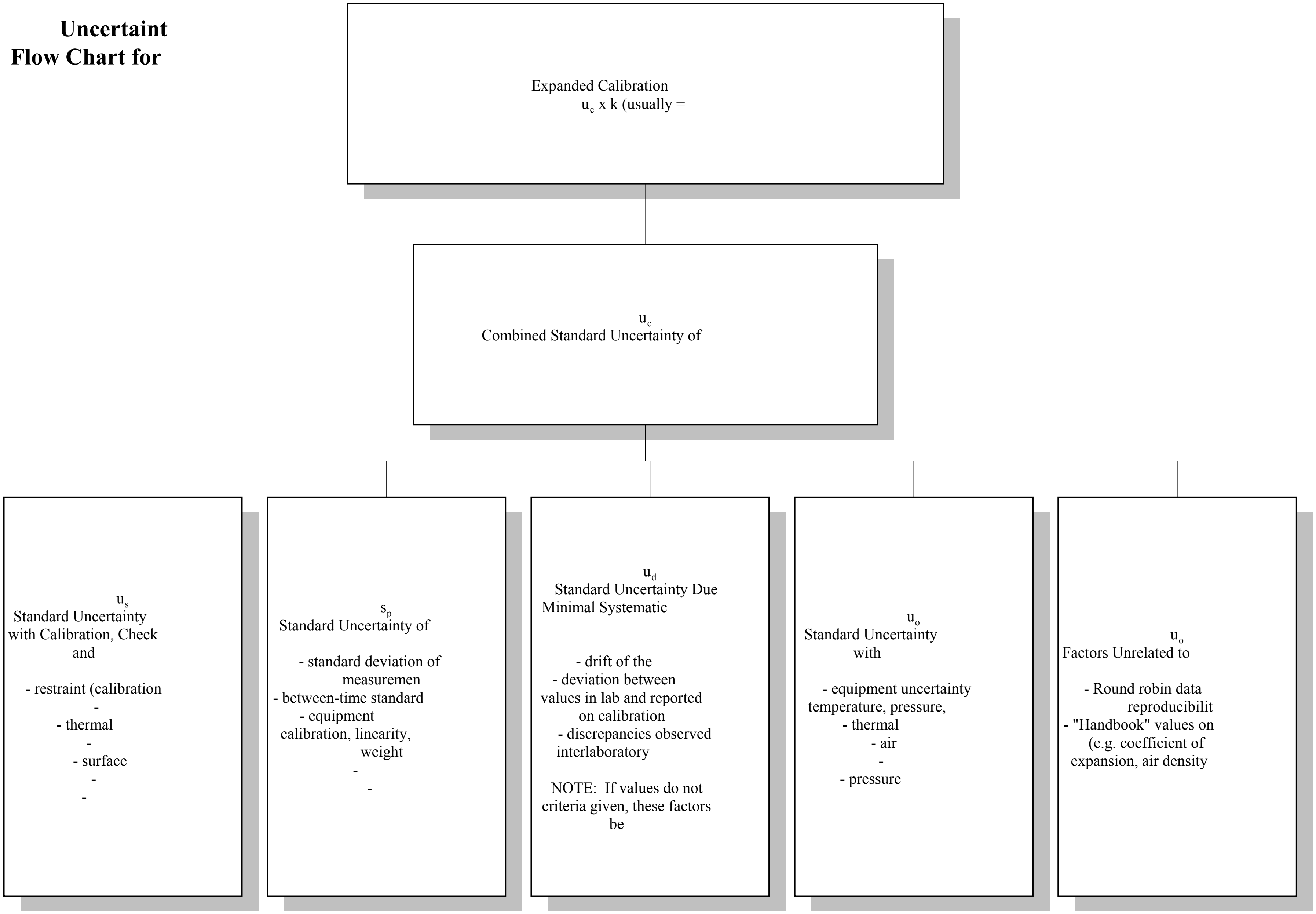
The expanded uncertainty may be evaluated against established criteria such as tolerance limits. For mass standards, the specifications clearly state that the expanded uncertainty must be less than 1/3 of the tolerance.

3.8. Step 8. Report the uncertainty

Once the uncertainty has been calculated, a statement such as the following is reported:

The combined standard uncertainty includes the standard uncertainty reported for the standard, for the measurement process, and for any observed deviations from reference (e.g., NIST) values, which are less than surveillance limits. The combined standard uncertainty is multiplied by *k*, a coverage factor of (2, 3) to give the expanded uncertainty (which defines an interval with an approximate (95, 99) % level of confidence).

Uncertaint
Flow Chart for



Uncertaint
Flow Chart for Length

Expanded Calibration
uc x k (usually =

u_c
Combined Standard Uncertainty of

u_s
Standard
Associated with
Check Standards and
Unknowns

- restraint (calibration report)
- uncertainty of optical scale
graph paper
- uncertainty of applied
- thermal
-
- surface
-
- hysteresis of
- readability of
-condition of tape

s_p
Standard Uncertainty
Process

- standard deviation of the
measurement (typically
range
- lighting
-
-

u_o
Standard
Associated with

- equipment uncertainty
temperatur
- thermal equilibrium
correction

u_d
Standard Uncertainty Due
Minimal Systematic

- drift of the
- deviation between
values in lab and reported
values on calibration reports
- discrepancies observed on
interlaboratory
- bias due to reading the
microscope or graph

NOTE: If values do not
criteria given, these
must be

u_o
Factors Unrelated to
Measuremen

- Round robin data showing
reproducibilit
- "Handbook" values on
standards (e.g. coefficient
thermal

Uncertaint
Flow Chart for