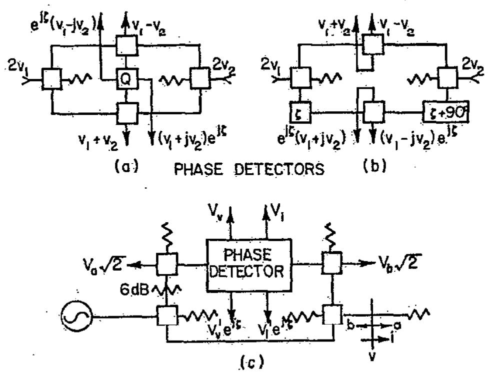

Fig. 6. Phase detectors from hybrid junctions or quadrature hybrids Q for getting $\sin\theta$ as well as $\cos\theta$.

tions having no matching transformers in the E and H arms usually have higher isolation, broader bandwidth, and are less expensive than matched hybrids. Note that either of the phase detectors of Fig. 6 can be made with unmatched hybrids. The impedance seen between any two adjacent hybrids in Fig. 6 is 25 or 100 Ω, but the impedance of all input and output ports is 50 Ω.

Conclusion

The six-port described here could be part of a broad-band automatic measurement system where component imperfections are corrected for by a computer that stores the frequency-sensitive corrections as derived from a suitable calibration program. The redundancy could provide a check on the accuracy. Analysis and application of a six-port for power measurements is given elsewhere in this issue [5].

Acknowledgment

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Application of an Arbitrary 6-Port Junction to Power-Measurement Problems

GLENN F. ENGEN AND CLETUS A. HOER

***Abstract*—An analysis of an arbitrary 6-port junction, four ports of which are terminated by power meters, shows that the net power input (or output) at either of the remaining ports is a linear function of the four power meter indications. The validity of this result has been experimentally demonstrated at 10 GHz. This device promises to be a useful tool in a wide variety of power measurement and calibration problems.**

I. Introduction

A DIRECTIONAL coupler with a sidearm power detector is a well-known form of power monitor. With the addition of a second coupler and detector, as shown in Fig. 1, the "reflected" as well as the "incident" power can be measured. In the most common mode, a signal source is connected to port 1, and the device serves as a feedthrough power meter for terminations of arbitrary impedance on port 2. Alternatively, with proper calibration, the device measures the input power at port 1 and may thus be interpreted as a variable impedance terminating power meter.

In order to improve the accuracy, tuning procedures have been devised which compensate for nonideal coupler performance (imperfect directivity, etc.) [1], [2]. In general, if the reflectometer of Fig. 1 is regarded as an arbitrary 4-arm junction, the power output P_2 at port 2 may be written [1]

$$P_2 = k_1P_4 - k_2P_3 + \alpha b_3 b_4^* + \alpha^* b_3 b_4^* \qquad (1)$$

where the real constants k_1, k_2, and the complex constant α are properties of the couplers; b_3, b_4 are the complex wave amplitudes at the respective detectors, $P_3 = |b_3|^2$, $P_4 = |b_4|^2$; and (*) denotes the complex conjugate.

As previously noted, it is possible to make α vanish by suitable tuning procedures, and P_2 is obtained in terms of P_3, P_4. Unfortunately, these methods tend to be both frequency sensitive and time consuming. Equation (1) indicates, however,

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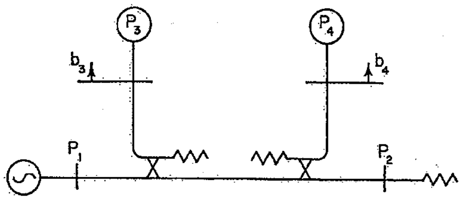

Fig. 1. 4-arm junction used in making power and impedance measurements.

that the tuning is unnecessary provided that the phase difference between b_3 and b_4 is known. Although today there is a significant trend in this direction, especially in automated systems, it must also be recognized that this represents a substantial increase in the complexity of the detection process since frequency conversion is generally involved. It thus becomes appropriate to investigate whether or not this phase detection requirement can be satisfied in terms of one or more additional amplitude measurements. The prior art includes the application of multiple detectors in the slotted line context [3], [4]. More recently, the application of an "ideal" 6-port to the measurement of the basic circuit parameters has been described [5]. The latter provided the incentive for finishing a partially complete and unpublished study which was begun several years ago in conjunction with the power equation concept [6]. To be more specific, this paper will consider the application of an *arbitrary* 6-port to power measurement problems. Moreover, the mathematical approach developed herein provides the basis for a significant generalization of the earlier 6-port effort. This topic, however, will be the subject of a future paper.

II. General Theory

For reasons which will emerge, it will prove useful to begin with an arbitrary 6-port, four ports of which are terminated by power meters, as shown in Fig. 2. It is convenient to use the complex incident and emergent wave amplitudes a_i, b_i, $i = 1 \cdots 6$, as the twelve terminal variables. The scattering equations may be written,

$$b_i = \sum_{j=1}^{6} S_{ij} a_j, \qquad i = 1 \cdots 6. \tag{2}$$

Moreover, the power meters on arms $3 \cdots 6$ are assumed to be "permanently" connected. Therefore,

$$a_j = b_j \Gamma_j, \qquad j = 3 \cdots 6 \tag{3}$$

where Γ_j is the reflection coefficient of the power meter terminating the jth arm. Equations (2) and (3) represent a collection of ten linear relationships among the twelve terminal variables. This system of equations may be solved for any ten of these as functions of the remaining two. In particular, it is possible to write:

$$b_i = r_i a_2 + t_i b_2, \qquad i = 3 \cdots 6 \tag{4}$$

where r_i, t_i are functions of the scattering parameters S_{mn} and Γ_j.

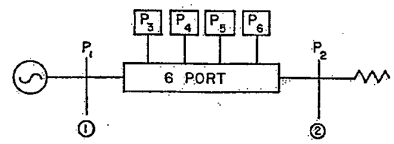

Fig. 2. Arbitrary 6-port junction.

Equation (4) is next multiplied by its conjugate to yield

$$P_i = |r_i|^2|a_2|^2 + r_i t_i^* a_2 b_2^* + r_i^* t_i a_2^* b_2 + |t_i|^2 |b_2|^2, \qquad i = 3 \cdots 6 \tag{5}$$

where $P_i = |b_i|^2$.

In a formal sense, (5) may be considered as a linear system of four equations in the unknowns $|a_2|^2, a_2 b_2^*, a_2^* b_2, |b_2|^2$ and may be inverted to obtain $|a_2|^2$ and $|b_2|^2$ as linear functions of the P_i ($i = 3 \cdots 6$). Finally, the difference between $|b_2|^2$ and $|a_2|^2$ will also be a linear function of the P_i so that[1]

$$P_{2\,(\text{net})} = |b_2|^2 - |a_2|^2 = \sum_{i=3}^{6} q_i P_i. \tag{6}$$

where the (real) q_i are functions (only) of the S_{mn} and Γ_j. Equation (6) is the desired result.

In deriving (6) the existence of a functional relationship between $a_2^* b_2$ and $a_2 b_2^*$ was ignored. This raises the question of whether it would be possible to write P_2 as a function of three of the power meters and eliminate the fourth. An investigation shows that this proposal leads to multiple roots and a considerable increase in complexity. For these reasons, this approach appears to be of limited, if any, practical interest and was not further explored.

III. Calibration Procedure

The object of the calibration procedure is to determine the q_i. Referring to Fig. 2, port 2 is first terminated by a standard power meter so that P_2 is known. The corresponding values of $P_3 \cdots P_6$ are then noted. This procedure then is repeated with the power meter replaced by three different offset shorts (or variable reactances), for which $P_2 = 0$. These values are now substituted into (6) to obtain four equations which may be inverted to obtain the q_i.

It should be noted that the standard power meter may be of arbitrary and unknown impedance. In addition, the arguments of the offset shorts are not required, although it is obvious that multiples of $\lambda/2$ must be avoided.

IV. Design Criteria

Although the foregoing procedures are valid for arbitrary 6-port junctions, there are singular cases which must be avoided. Although some of these are intuitively obvious, there are other cases for which this is not true. For this reason it is desirable to develop some design criteria for the 6-port.

Returning to Fig. 1 and (1), the magnitude of α is one measure of the deviation of the couplers from the ideal. Ordinar-

[1] The normalization factor has been taken equal to unity.

ily, $|\alpha|$ is a small number. In addition, P_3, P_4 are approximately proportional to $|a_2|^2$, $|b_2|^2$. It thus appears desirable to begin with the system of Fig. 1 and look for a modification which would permit the determination of the terms involving α.

By eliminating a_2, b_2 from (4) it is possible to write:

$$b_5 = Kb_3 + Lb_4$$
$$b_6 = Mb_3 + Nb_4 \qquad (7)$$

where K, L, M, N are functions of S_{mn} and Γ_j. If each of (7) are multiplied by their conjugate, it is possible to obtain

$$KL^*b_3b_4^* + K^*Lb_3^*b_4 = P_5 - |K|^2P_3 - |L|^2P_4 \qquad (8)$$

$$MN^*b_3b_4^* + M^*Nb_3^*b_4 = P_6 - |M|^2P_3 - |N|^2P_4. \qquad (9)$$

Equations (8) and (9) may now be solved for $b_3b_4^*$ and $b_3^*b_4$. If this is done, and the results substituted in (1), an equation in the form of (6) is obtained (which, of course, is to be expected). In order for this procedure to be well conditioned, it is necessary to avoid the case where the determinant Δ of the coefficients in (8) and (9) is small. This determinant is given by

$$\Delta = KL^*M^*N - K^*LMN^* \qquad (10)$$

which indicates that KL^*M^*N is preferably an imaginary number, and in no case is permitted to become real.

To see what this means in practical terms it is useful to consider specific implementations, including some which fail to satisfy these criteria. In Fig. 3 a portion of the signals b_3, b_4 have been combined in a hybrid to yield outputs proportional to $b_3 + b_4$, $b_3 - b_4$. For this combination one has $K = L = M = -N$, and the above criteria are violated ($\Delta = 0$). Instead of the $b_3 - b_4$ term, the system should provide $b_3 \pm jb_4$. A 6-port which satisfies these criteria is shown in Fig. 4. An alternative configuration is shown in Fig. 5. Here the two couplers have been supplemented by two voltage probes with a $\lambda/8$ spacing. This $\lambda/8$ spacing represents a design center, and in practice, spacings between the limits $\lambda/24$ to $5\lambda/24$ should prove satisfactory. Assuming adequate coupler bandwidth, this should permit operation over at least a 5-1 frequency ratio. In practice it may prove possible to extend these limits and achieve a 10-1 bandwidth.

In practical terms a convenient (but not most general) set of design goals are

$$b_3 \simeq e\,a_2$$
$$b_4 \simeq f\,b_2$$
$$b_5 \simeq g(a_2 + b_2)$$
$$b_6 \simeq h(a_2 + jb_2) \qquad (11)$$

where e, f, g, and h are a set of complex parameters whose arguments are arbitrary and whose magnitudes determine the signal levels at the respective detectors.

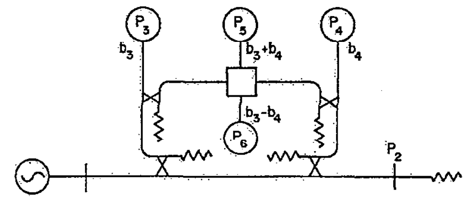

Fig. 3. Example of how not to make a 6-port for power measurements.

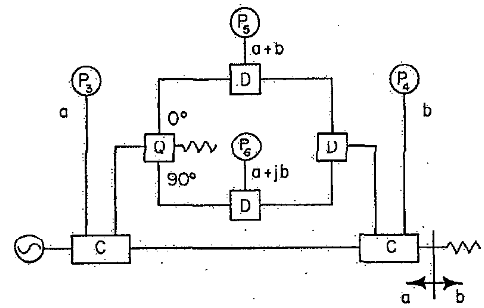

Fig. 4. 6-port junction designed to approach the ideal output conditions for measuring power, where C is a 4-port coupler, hybrid junction, or quadrature hybrid, Q is a quadrature hybrid or a 3-dB quadrature coupler, and D is a power divider or hybrid junction.

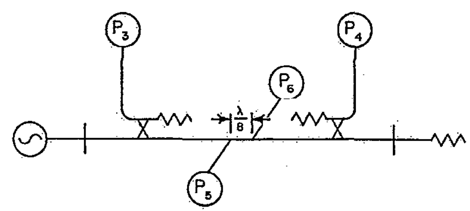

Fig. 5. Proposed 6-port junction for use in waveguide.

V. Applications

It should be noted that (6) gives the power at port 2, not only with the generator connected as in Fig. 2, but also with *the positions of the generator and load exchanged.* (P_2 *is now* negative.) Moreover, the power input or output at port 1 is also a linear function of $P_3 \cdots P_6$.

Because of its flexibility, this device should prove useful in a wide variety of power measurement problems. An immediate application is that of calibrating bolometer mounts or other types of power meters.

VI. 6-Port Versus Reflectometer

To compare power measurements using a 6-port with power measurements using a generalized reflectometer (g-reflectometer), the circuit shown in Fig. 6 was assembled using X-band waveguide components. This composite circuit includes the necessary elements to permit a power measurement either by the 6-port technique or by the tuned g-reflectometer procedure. Moreover, the circuit is so designed that the coupler

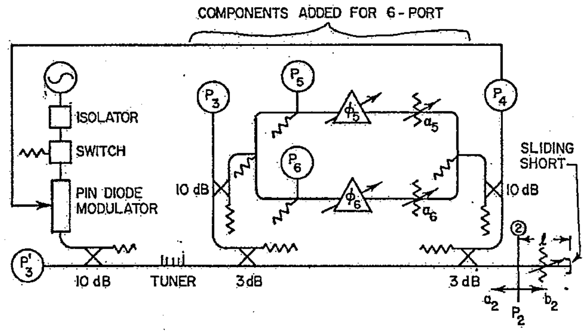

Fig. 6. Experimental X-band setup for comparing power measurements made with a 6-port to power measurements made with a reflectometer.

which provides the dominant term P_4 is common to both implementations. To be more specific, that portion of the circuit to the right of the tuner in Fig. 6 represents an implementation of the 6-port concept. Alternatively, the presence of the detectors P_3, P_5, and P_6 may be ignored and attention focused on P_4 and P_3', their associated directional couplers, and the tuner. These elements comprise a g-reflectometer whose operation is described in [1].

The components ϕ_5, α_5, ϕ_6, and α_6 were adjusted so that $P_5 = P_6 = 0$ when a short is at terminal 2. Then ϕ_6 was increased 90°. This gives the desired condition for the 6-port, namely,

$$P_5 = c|a_2 + b_2|^2, \quad P_6 = c|a_2 + jb_2|^2$$

where c is some unknown constant. Then, with a sliding short connected at port 2, the tuner was adjusted for no variation in P_3' as the position of the short was changed. This adjustment makes $\alpha \simeq 0$ in (1) so that P_2 measured by the g-reflectometer is given by

$$P_2|_{\text{refl}} = k_1 P_4 - k_2 P_3'.$$

The last two terms in (1) are zero only for the g-reflectometer, not the 6-port. The 6-port must evaluate these last two terms. Since both systems use the same P_4, and both P_3 and P_3' are approximately the same, each system has the same sensitivity. Both systems measure power at reference port 2, so that P_2 measured by the 6-port should be identical to P_2 measured by the g-reflectometer.

After calibrating the g-reflectometer to get k_1 and k_2 and calibrating the 6-port to get the four q_i, the net power into a variable impedance load was measured by both systems at 10 GHz. The variable impedance load was an attenuator followed by a sliding short as shown in Fig. 6. For each setting of the attenuator (1, 3, 7, and 10 dB), the net power to the load was measured at four different positions of the short spaced approximately $\lambda/8$ apart. The results are shown in Table I.

The last column in Table I shows the percent deviation of P_2 as measured by the g-reflectometer from P_2 as measured by the 6-port, relative to the incident power which was 8.4 mW. As one can see, the agreement is better than 0.25 percent when $|\Gamma|$ was 0.2 or less. At larger values of $|\Gamma|$, the difference is somewhat greater. This is explained by noting that as $|\Gamma|$ increases, the terms P_3', P_3, P_5, P_6 become larger, and errors in their measurement are of greater significance. Because the instrumentation used to measure those terms provides only a nominal recovery of 3 percent, the agreement is still within the experimental error. In practice $|\Gamma|$ seldom exceeds 0.2; however, the performance at larger values of $|\Gamma|$ yields a more convincing demonstration of the validity of (6).

TABLE I
NET POWER MEASURED BY A 6-PORT COMPARED TO THE SAME POWER MEASURED BY A REFLECTOMETER INTO A LOAD OF REFLECTION COEFFICIENT Γ

$\|\Gamma\|$	$P_2\|_{\text{6-port}}$	$P_2\|_{\text{refl}}$	$P_2\|_{\text{refl}} - P_2\|_{\text{6-port}}$	Percent of P_I (P_I = 8.40 mW)
0.8	1.078	1.195	+0.117	1.39
0.8	1.229	1.196	−0.033	0.39
0.8	1.016	1.042	+0.026	0.31
0.8	1.208	1.123	−0.085	1.01
0.5	5.438	5.510	+0.072	0.86
0.5	5.492	5.564	+0.072	0.86
0.5	5.434	5.535	+0.101	1.20
0.5	5.496	5.541	+0.045	0.54
0.2	7.946	7.950	+0.004	0.05
0.2	7.943	7.949	+0.006	0.07
0.2	7.913	7.933	+0.020	0.24
0.2	7.967	7.965	−0.002	0.02
0.1	8.298	8.298	0.000	0.00
0.1	8.289	8.294	+0.005	0.06
0.1	8.277	8.286	+0.009	0.11
0.1	8.314	8.310	−0.004	0.05

VII. SUMMARY

In the prior art, the elimination of errors due to imperfect coupler directivity, etc., have been eliminated at the price of elaborate tuning procedures or phase detection schemes. As previously noted, the former tends to be both frequency sensitive and time consuming, while the latter generally involves frequency conversion and usually places substantial demands upon the frequency stability of the source. The measurement scheme outlined in this paper eliminates both of these requirements but at the expense of two additional amplitude measurements.

Because substantial amounts of data are involved and a matrix inversion is required in the calibration procedure, a certain amount of computer-aided computational facility is highly desirable. On the other hand, because P_2 is a linear function of $P_3 \cdots P_6$, it is possible to envision a simple analog circuit whose parameters could be adjusted with changes in frequency, which would yield P_2.

Finally, it may be noted that the theory begins with an arbitrary 6-port and the subsequent calibration makes no reference to matched loads or the arguments of the reflection from the shorts. In addition, only *net* powers are involved. The procedure thus satisfies the terminal invariant criteria and belongs to the family of power equation methods [6].

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Microwave Peak Power Measurement Independent of Detector Characteristics by Comparison of Video Samples

FRITZ K. WEINERT AND BRUNO O. WEINSCHEL

Abstract—For measurement of RF pulse power, a 100 percent square-wave-modulated reference RF power is interlaced with the pulses. 100-ns-long samples are taken after detection from both the demodulated pulse and square wave. The level of the reference power is adjusted until both samples are equal. This method is independent of the temperature coefficient, law or stability of the detection efficiency. The method is compared with the notch wattmeter, crystal video-detector chopper, and sampling comparison using crystal switches.

I. Definition of a Pulse

THE envelope of an RF pulse as detected by a perfect linear detector appears in Fig. 1. Rise and fall time is measured between the 10- and 90-percent voltage points. Pulse length is measured between the 50-percent voltage points. For measurement of pulse power, one usually disregards the first and last 100 ns. In this method the peak power is defined as the average rms power in the center portion of the pulse. This is actually a refinement of the old IRE definition of "peak-pulse amplitude," which is "the maximum absolute peak value of the pulse, excluding those portions considered to be unwanted, such as spikes. Note: where such exclusions are made, pictorial illustration of the amplitude chosen is desirable" [9].

The peak-to-peak ripple within that portion and the peak-to-peak jitter must be small if the pulse is used for calibration. 2 percent and 25 ns are good practical limits. When an unknown pulse is measured, both may be worse, which may limit the measurement accuracy.

Only rectangular pulses repeated at a fixed frequency are treated. The method also applies to a Gaussian pulse as used in TACAN. It can be readily adapted to multiple pulses or pulses with staircase envelopes by redefining pulse power and sampling location.

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II. Fidelity of Detected Pulse

A detected pulse trails behind the RF pulse by a time delay

$$T_D = RC \tag{1}$$

where R is approximately the resistance of the parallel combination of the diode load resistor and the detector video resistance while C is the output capacity to ground. Fig. 2 shows this lag.

The rise time of the microwave detector must be sufficiently short so that the video output voltage nearly reaches its correct value when the sampling period starts. This requirement is most critical for the shortest pulse duration of 350 ns. At the time when the sampling gate opens, the video output has not reached the correct voltage E_{max} yet, causing a difference voltage of $\delta \cdot E_{max}$.

In order for the measurement not to contain an appreciable error, δ must be below 0.01 dB. The results of a numerical evaluation are shown in Fig. 3 for linear and square law detectors. The distortion in Fig. 2 is based on a time constant of 12 ns. This time constant results from an internal resistance of 5000 Ω shunted by 1000 Ω.

III. Temperature Dependency of Detection Efficiency of Microwave Diode Detectors

A typical microwave detector equivalent circuit is shown in Fig. 3 where Z_0 is the RF termination, required for good VSWR and R_L the load resistor required to achieve fast rise