WR 10 Millimeter Wave Microcalorimeter

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Issued June 1981

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A microcalorimeter has been built in WR 10 waveguide, 75-110 GHz, to serve as a power standard at the National Bureau of Standards (NBS). Included here is an evaluation of the errors in using the microcalorimeter for the measurement of effective efficiency of bolometer mounts.

The error analysis shows a systematic uncertainty of ± .83 percent and a random uncertainty of .37 percent.

Key Words: calorimeter; millimeter wave; power; standard.

1. INTRODUCTION

The standards for waveguide power measurement at the National Bureau of Standards (NBS) in the microwave and millimeter wave region are a series of microcalorimeters which evolved from the work of MacPherson and Kerns [1], Engen [2], and Harvey [3]. This paper describes the latest addition to the series, the WR 10 waveguide unit which is identical in design and construction to the WR 15 microcalorimeter developed by Harvey [3]. The only difference between the WR 10 and WR 15 models is the waveguide size and the thermistor mounts used as working standards.

Details of construction (and operation) of the microcalorimeter are not included in this paper since they are described in reference [3].

The main purpose of this paper is to document the evaluation of uncertainties in the WR 10 microcalorimeter. All of the work was done in a two percent band centered at 95 GHz.

2. PRINCIPLE OF OPERATION

A complete derivation of the operating principle of the microcalorimeter is given in reference [2]. However, for the sake of completeness, a brief summary is given below.

While the NBS microcalorimeter is not a true calorimeter (i.e., it measures rate rather than total quantity of heat), the calorimetric principle is nevertheless used to determine the effective efficiency of bolometric detectors such as waveguide thermistor mounts. The latter consist of a thermistor (temperature sensitive resistor) mounted in a shorted section of waveguide, the combination forming a terminating load to the incident microwave or millimeter wave energy. Connections to the sensing thermistor are made via a low pass filter so that it becomes, for example, one arm of a Wheatstone bridge. When dc power is applied to the bridge (and thus to the thermistor) the power absorbed by the thermistor causes its temperature to increase and its resistance decreases (negative temperature coefficient). Thus, by adjusting the dc power, the bridge may be brought to a balanced condition and the thermistor resistance (typically 200Ω) is determined by the bridge parameters. In recent years, self-balancing circuits have been developed which automatically keep the bridge balanced. With negligible error, it can be assumed that all of the dc power input to the mount is absorbed by the thermistor bead.

When mm wave power is supplied to the thermistor, the self-balancing circuit automatically withdraws a portion of the dc bias power to maintain the bridge in balance. However, some of the input mm wave energy does not reach the thermistor bead, instead it is absorbed in the mount structure. Thus, the dc power withdrawn from the thermistor bead is smaller than the rf input power to the mount since only that energy which actually reaches the thermistor contributes to its heating. This leads to the concept of mount efficiency, η, which is less than unity. A second problem is the so-called rf-dc substitution error which is due to differences in the heating effect on the thermistor for equal amounts of RF versus dc power. This is equivalent to a further change in mount efficiency but may be either positive or negative. The microcalorimeter measures the combination of the two effects which is defined as effective efficiency, η_e.

The heat generated in the thermistor bead is conducted, convected, and radiated to the mount structure so that the mount temperature, T, rises above the ambient, T_o. With dc power only in the thermistor, this temperature will be T_1. When mm wave power is applied to the mount, I^2R and dielectric losses in the mount structure generate additional heat and the temperature of the structure rises above T_1 to T_2. The difference between T_2 and T_1 is related to the difference between η_e and unity.

It is the above effects that the microcalorimeter is designed to detect and measure. The principle components of the calorimeter are a waveguide flange which mates with the thermistor mount to be measured, a metal reference ring which surrounds this flange but is thermally isolated from it and a 47 junction copper-constantan thermopile connected between the two. The thermopile generates voltages e_1 and e_2 which are proportional to T_1-T_0 and T_2-T_0, respectively.

The bolometer units used as working standards in the WR 10 system are commercially available thermistor mounts. No modification was required of the commercial design in contrast with the WR 15 mounts, which were also commercial units but NBS modified to reduce mm wave leakage on the dc leads. Instead, the WR 10 mounts have been improved by the manufacturer to reduce millimeter wave leakage by an order of magnitude (0.1 to 0.2%) and the resultant design is actually an improvement over the WR 15 mounts. The WR 10 working standard thermistor mounts have a shorter input waveguide lead and better thermal coupling to the calorimeter flange; they also have a lower reflection coefficient than the WR 15 mounts.

3. ERROR ANALYSIS

The equation used to calculate the effective efficiency (η_e) of the working standard thermistor mount using the microcalorimeter is [2]

$$\eta_e = \frac{g[1 - (V_2/V_1)^2]}{e_2/e_1 - (V_2/V_1)^2}, \tag{1}$$

where V is the bridge voltage used for biasing the thermistor to its operating resistance and e is the thermopile output voltage. Subscripts 1 and 2 in equation (1) denote mm-wave (millimeter wave) power OFF and ON respectively. The term g is an algebraic correction for systematic effects and was defined in reference [2].

In order to obtain a value for the systematic correction g, equation (3) in reference [2]

$$e_2 = k(P_{2dc} + gP_{rf}) \tag{2}$$

can be expanded to

$$e_2 = k(P_{2dc} + aP_{rfb} + bP_{rfw} + cP_{rff} + dP_{rfi}) \tag{3}$$

In equation (3), the coefficients a...d are the relative effectiveness of heating from sources, other than the thermistor bead, on the thermopile response as compared to heating from the thermistor bead and power dissipation in its immediate vicinity. Here, P_{2dc} is the dc power in the bead with mm wave power applied, P_{rfb} the mm wave power dissipated in the bead and its immediate vicinity, P_{rfw} the mm wave power dissipated in the bolometer unit walls, P_{rff} the mm wave power dissipated at the calorimeter-bolometer flange interface, and P_{rfi} the mm wave power dissipated in the thermal isolation and flange waveguide sections.

It is assumed that mm wave heating in the vicinity of the thermistor bead has the same effect on e_2 as P_{2dc} so that a=1.

The mm wave powers, P_{rfb}, P_{rfw}, P_{rff}, P_{rfi} can be expressed as percentages of P_{rf} (the total mm wave power dissipated in the bolometer unit). Comparing equations (2) and (3)

$$gP_{rf} = aP_{rfb} + bP_{rfw} + cP_{rff} + dP_{rfi} \tag{4}$$

and

$$P_{rfb} = qP_{rf}$$
$$P_{rfw} = rP_{rf} \qquad (5)$$
$$P_{rff} = sP_{rf}$$
$$P_{rfi} = tP_{rf}$$

where q...t represent that fraction of the total mm wave power dissipated in the bolometer mount which is dissipated in the various places as outlined in the above definitions of $P_{rfb} \ldots P_{rf}$. In this case

$$q + r + s = 1 \qquad (6)$$

assuming P_{rfb}, P_{rfw}, and P_{rff} account for all power dissipated within the bolometer mount. (Note that P_{rfi} is dissipated <u>outside</u> the bolometer mount). Using equations (4) and (5)

$$g = aq + br + cs + dt \qquad (7)$$

The factors b...d and q...t are evaluated in the following sections.

3.1 Bolometer Unit Walls and Flange

A given quantity of millimeter wave energy absorbed (i.e., converted into heat) in the thermistor mount between the input flange and the thermistor bead will have a greater effect on the thermopile output than the same quantity of energy absorbed in the thermistor bead. The systematic correction which compensates for this effect is contained in the terms cs and br in equation (7).

In order to evaluate this correction, measurements were made with a heat source (small dc heater with input power, P_1) at the microcalorimeter-thermistor mount flange interface. This showed that the heat source at the flange (worst case condition) had an approximately one percent greater effect on the thermopile output than the same amount of power, P_1 in the thermistor bead. The difference in thermopile response between the two locations for the heat source is small owing to the large thermal conductivity of the thermistor mount. The short input waveguide lead (9 mm long) and the thermal coupling through the mount via an equivalent metal, thick-walled (25mm diam.) cylinder accounts for the small difference between heat source locations.

Using flange loss measurements in the microwave and millimeter wave region up to 60 GHz, an extrapolated value of 0.3 percent of the input mm wave power (P_{rf}) was obtained for flange loss at 95 GHz. These ts lead to the values c = 1.01 and s = 0.003 in equation (7).

A combination of theory and extrapolation from WR 15 waveguide loss measurements at 60 GHz was used to arrive at a figure of 0.0039 dB/mm for waveguide loss at 95 GHz. In the 9 mm distance between flange and thermistor bead this would lead to the estimate that 0.035 dB or 0.8 percent of the input power is dissipated in the input guide, or r = 0.008 in equation (7). Assuming uniform distribution of the loss, an estimate of 1/2 times the increased effect of heating at the flange would be appropriate. This implies that b = 1.005 in equation (7) for input guide wall losses.

3.2 Thermal Isolation Section

The thin-wall thermal isolation waveguide section which terminates in a flange forms a 9 mm long input waveguide ahead of the bolometer-microcalorimeter flange interface. The loss in this thin-wall section which includes the distance through the flange "thickness" contributes to the systematic uncertainty. Based on the microcalorimeter geometry, an estimated 75% of the associated heat is conducted and convected to the flange causing an increase in e_2. Since this source of heat is outside the bolometer mount, a correction must be applied. A value for this correction is arrived at using the same arguments as for the bolometer unit wall loss in the previous section.

The power dissipation in a 9 mm length waveguide section, already calculated in section 3.1, is 0.8 percent and 75 percent of this yields 0.006 for the value of t in equation (7). For d in equation (7) the value of 0.5 percent is used for the increased heating effect in this region (one-half the effect at the flange). This means that d in equation (7) is 1.005.

3.3 Calculation and Uncertainty for g

From the preceding sections, r = 0.008 and s = 0.003, and from equation (6) this implies q = 0.989 or 98.9 percent of the mm-wave power dissipated in the bolometer unit is dissipated in the vicinity of the thermistor bead.

Using the preceding values for a...d and q...t and equation (7) yields g = 1.0061. If it is assumed that the contributions to g are in error by ±100 percent (i.e., $1 \leq g \leq 1.0122$) the uncertainty in g is ±0.61 percent. The ±100 percent uncertainty is probably conservative, but waveguide and flange losses and heating effects are difficult to measure in the mm-wave frequency range, and a cautious approach is warranted.

A summary of these results is listed in Table I.

3.4 Millimeter Wave Leakage Effects

Millimeter wave leakage from the bolometer mount via the dc bias leads causes a systematic error in the measurement of η_e in the microcalorimeter. Since η_e is defined as P_{dc}/P_{rf} where P_{dc} is the substituted dc power and P_{rf} is the total mm-wave power dissipated in the bolometer unit, the leakage power as a percentage of P_{rf} should be a correction to η_e as measured in the microcalorimeter. When the bolometer is used outside of the microcalorimeter, it is assumed to measure all of the mm-wave power at its input flange. The correction can be seen by relating two values for η_e as follows:

$$\eta_e = P_{dc}/P_{rf} \tag{8}$$

not accounting for leakage and

$$\eta_{e\ell} = \frac{P_{dc}}{P_{rf} + P_\ell} \tag{9}$$

where P_ℓ is the leakage power.

Then if

$$P_\ell = uP_{rf} \tag{10}$$

where u is the fraction of leakage power and using equations (8)-(10)

$$\eta_{e\ell} = \frac{\eta_e}{1 + u} \tag{11}$$

The leakage measurement was made by changing the orientation of the dc biasing leads to the thermistor with millimeter wave power applied at the waveguide input and noting the percentage change in substituted dc power. This measurement showed an effect of approximately ±0.02 percent which suggests that at least 0.02 percent of the input power was leaking out via the dc leads. Since this measurement gives only an indication of actual leakage, this result is multiplied by a factor of 10 for estimating leakage with an uncertainty of ±100%. In equation (11) (1 + u) = 1.002 ±0.002.

The leakage correction is listed in Table I.

3.5 Instrumentation

Since the bridge voltages and thermopile voltages occur in ratios in equation (1) there is no significant contribution to systematic error. Instrumentation effects will appear in the overall random error.

3.6 Thermopile Nonproportionality

Calculation of η_e from equation (1) assumes that the ratio, e_2/e_1, is linear with power ratio, P_2/P_1, where P_2 is the power absorbed in the thermistor mount with mm wave power plus dc and P_1 is the absorbed power with dc only. Linearity of e_2/e_1, is assured if it can be shown that e_1 and e_2 are individually linear with the temperature difference between the thermopile hot and cold junctions, T, and that T varies linearly with P.

Thermocouple output, e, is, in fact, somewhat nonlinear (increasing) with respect to T. Also, nonlinear cooling due to radiation causes T to be smaller than would be the case for linear cooling (i.e., conduction). It is shown in the appendix, however, that these nonlinear effects on e_2/e_1 are negligible.

3.7 Thermistor Radiation and Convection

As described in [3], this is a small contribution to the systematic error (0.001 percent). The WR 10 mounts have a white, low-loss dielectric thermal barrier in the waveguide so that radiation and convection of heat out of the bolometer mount are reduced and have even less effect.

3.8 Imprecision

Effects such as instrumentation error, temperature instability, and flange non-repeatability are all included in the estimated value for random error.

The combined long-term and short-term random uncertainty was estimated from repeat measurements on the two working standard bolometers at 94.0, 94.5, and 95.0 GHz, each on three occasions. The pooled value for the standard deviation of the mean for either working standard bolometer at any of the above frequencies was calculated to be 0.10%.

The estimate for random error has approximately 6 degrees of freedom leading to a t value of 3.707 at the 99% confidence level. Therefore, the random error at the 99% level is ±0.37%.

4. SUMMARY

The systematic corrections and their uncertainties are listed in Table 1. The total estimated systematic correction, $\frac{g}{1+u}$, is 1.0041.

The sum of systematic and random uncertainties is ±1.2 percent for effective efficiency as compared to 0.25 to 0.30 percent for the microcalorimeters in the larger waveguide sizes WR 15, WR 28 and WR 42.

Table 1

Systematic Corrections and Limits of Uncertainty

$$\eta_e = \frac{g}{1+u} \quad \frac{[1 - (V_2/V_1)^2]}{e_2/e_1 - (V_2/V_1)^2}$$

	Correction	Systematic Uncertainty
Thermopile Nonproportionality	0.0000	± 0.02%
g = ag + br + cs + dt		
= 1 (.0.989) + 1.005 (0.008) +		
1.010 (0.003) + 1.005 (0.006)	1.0061	± 0.61%
Millimeter Wave Leakage (1 + u)	1.0020	± 0.20%
Thermal Leakage	0.0000	± .001%
Totals	1.0041	± 0.83%

5. REFERENCES

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[4] Souders, Mott; The Engineers Companion, John Wiley and Sons, New York, 1966, p. 197.

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Appendix

Analysis of Thermopile Nonproportionality

Nonlinear cooling effects on the flange in the microcalorimeter are due mainly to thermal radiation. As mentioned previously, when dc power, P_1, is applied to the thermistor mount being measured, the equilibrium temperature of the flange rises from T_0, the bath temperature, to T_1. This temperature difference gives rise to a thermopile output, e_1. When mm wave power is substituted for a portion of the dc power, the total power absorbed in the thermistor bead and mount increases to P_2, the flange temperature rises to T_2 and the thermopile output increases to e_2.

Since it is required that e be a linear function of the dc and mm wave powers, the analysis will be carried out in two steps. First will be calculation of the nonlinearity of e with respect to T and second the relationship between T and P.

A. Nonlinearity of e with respect to T.

For this analysis, only a single thermocouple will be considered because of the linear relationship between the output voltage of thermocouples and thermopiles.

The response, e, of a copper-constantan thermocouple as a function of ΔT, the temperature difference between the hot and cold junctions, is shown below (NBS Circular 508).

ΔT (°C)	e(μV)	$e/\Delta T$ (μV/°C)
1	38	38.0
10	389	38.9
20	787	39.35
30	1194	39.80

As can easily be seen, the response is nonlinear. The model for the response is closely approximated by the empirical equation

$$e = 37.91(\Delta T) + 0.09(\Delta T)^2 - 0.009(\Delta T)^3$$

The temperature rise, ΔT, of the flange (hot junctions) relative to the reference ring (cold junctions) is approximately 0.05K. Substituting this value into the above equation

$$e = 37.91\ (0.05) + (0.09\ (0.05)^2\ \ (0.009)\ (0.05)^3$$

$$= 1.895 + 0.00023 - 0.000001$$

The nonlinear fraction of e in percent is given by

$$\text{nonlinear percent} = \frac{0.00023}{1.895} \times 100 = 0.012$$

Thus, in assuming that the thermopile output, e, is linear for a temperature difference of approximately 0.05K, an error of 0.012 percent results.

B. Analysis of Nonlinear Cooling Effects

At thermal equilibrium, the rate of heat loss from the flange is equal to the rate of heat input which, in turn, is proportional to power, P. If the heat loss were due entirely to conduction, then linearity between flange temperature, T, and P would be assured. However, other heat loss mechanisms exist, namely convective, $K_1 \Delta T^{5/4}$, and radiative, $K_2(T_1^4 - T_0^4)$ where T_1 and T_0 are absolute temperatures.

Calculating the convective heat loss [4]

$$q = h\ A\ \Delta T \text{ watt/m}^2\text{K}$$

For air in laminar flow around a vertical cylinder

$$\begin{aligned} h &= 0.29(\Delta T/L)^{1/4} \\ &= 0.29\left(\frac{.025}{.01}\right)^{1/4} \\ &= 0.29\ (2.5)^{1/4} \\ &= 0.365 \end{aligned}$$

and

$$\begin{aligned} q &= 0.365 \left[\pi \frac{(.02)^2}{4}\right] 0.025 \\ &= 9.12 \times 10^{-3} \times 3.14 \times 10^{-4} \\ &= 2.87 \times 10^{-6} \\ &\simeq 3\mu W \end{aligned}$$

The total power absorbed by the thermistor bead and mount structure is approximately 25 mW (25,000 μW) and thus convective cooling is of the order of 0.012 percent of the total.

Turning now to calculation of cooling by radiation, we use the Stefan-Boltzmann law for net radiated power loss, E_b [5].

$$E_b = \varepsilon\tau(T^4 - T_0^4)$$

where $\tau = 5.67 \times 10^{-12}$ watts/cm^2-$^oK^4$, and ε is the emissivity of the surfaces relative to a black body. The value of ε for bright metal surfaces is in the range 0.030 to 0.047.

The value of T_0 for our microcalorimeter is approximately 300 K while T assumes the values 300.05 K (dc power only) and 300.055 K (dc plus mm wave power). We will calculate the rate of total radiative heat loss and show that it is less than 0.01 percent of conductive heat loss and thus nonlinearity of radiative heat loss is negligible.

The rate of maximum radiative heat loss is given by

$$E_b = 0.047 \text{ a } 5.67 \times 10^{-12} \; (300.055)^4 - (300)^4$$

$$= 2.67 \times 10^{-13} \times 10^8 \; (81.059 - 81.0)$$

$$= 1.58 \times 10^{-6} \text{ W}$$

$$= 1.58 \; \mu\text{W}$$

Since the total heat loss rate is 25 mW (25,000 μW), the fraction of radiative heat loss is $1.58/2.5\text{x}10^4$, or 0.0063 percent.