Measurements of the Characteristic Impedance of Coaxial Air Line Standards

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Abstract— A method for electrically measuring the characteristic impedance of coaxial air line standards is described. This method, called the gamma method, determines the characteristic impedance of a coaxial air line from measurements of its propagation constant and capacitance per unit length. The propagation constant is measured on a network analyzer, and the capacitance per unit length is measured on a capacitance bridge at 1 kHz. The measurements of characteristic impedance with the gamma method are independent of any dimensional measurements. Measurements of the characteristic impedance using the gamma method are compared to theoretical predictions from dimensional measurements. Test results are shown for 14 mm, 7 mm, and 3.5 mm coaxial air lines.

I. INTRODUCTION

TODAY's modern microwave network analyzers rely on air linestandards for the accuracy of their measurements. Air lines are used both to calibrate a network analyzer and to verify their performance after calibration. One of the properties of a coaxial air line that is of interest to microwave engineers is its characteristic impedance. This paper describes the measurements of characteristic impedance of some beadless coaxial air lines.

Historically, the characteristic impedance of a coaxial airline has been determined from dimensional measurements. Air gauging techniques are normally used to measure the diameters of both the inner and outer conductors. The diameter is determined by measuring the small changes in air pressure as a probe moves inside the bore of the outer conductor or over the outside of a center conductor. Currently, air lines are measured at NIST with air gauge techniques to accuracies of 1 μm (40μin) or better.

One of the problems with air gauging is that it does not work at the ends of the air lines due to the loss of air pressure. Dimensional problems that occur at the ends of air lines can go undetected. Another problem with the dimensional characterization of air lines is that the effective resistivity of the metal must be known in order to accurately predict the characteristic impedance. The effective resistivity of the metals is often not known, particularly in the 10 MHz to 1 GHz range where skin depth penetration is significant. Surface roughness, metal impurities, and the quality of plating can increase the effective resistivity of the metals.

In this paper, we describe the gamma method for measuring the characteristic impedance of a coaxial air line. This method was originally developed to measure the characteristic impedance of MMIC standards [1], [3]. With the gamma method, the characteristic impedance is determined from measurements of the air lines propagation constant, and the capacitance per unit length. The gamma method differs from the conventional dimensional method in that the characteristic impedance is determined entirely from electrical measurements. No estimate of the effective resistivity is required. With the gamma method, the average characteristic impedance is measured, in contrast to the dimensional method where the characteristic impedance is determined along the length of the air line. The gamma method provides a means for verifying the characteristic impedance of an air line independent of dimensional measurements. This paper compares results using the gamma method to predictions from conventional dimensional metrology. Measurements are shown for 14 mm, 7 mm, and 3.5 mm beadless, coaxial, air lines.

II. THEORY

A. Gamma Method

The characteristic impedance of a coaxial air line is given by

$$Z_0 = \frac{\gamma}{j\omega C + G} \qquad (1)$$

where γ is the complex propagation constant, ω the radian frequency, C the capacitance per unit length, and G the conductance per unit length of the transmission line [1]. For an air dielectric, coaxial air line, G is negligible compared to wC, so that (1) can be approximated as

$$Z_0 = \frac{\gamma}{j\omega C}. \qquad (2)$$

The propagation constant γ can be measured with a commercialnetwork analyzer. The quantity γD, where D is the length of the air line, is obtained when an air line is measured in a normal TRL (thru-reflect-line) calibration [2]. A complete calibration of the network analyzer is not required since γD can be obtained from only the thru and line measurements.

Measurements were made on 14 mm, 7 mm, and 3.5 mm air lines. The 14 mm and 7 mm connectors are sexless and a true thru connection is possible. A true thru connection is where the test ports are connected together. The 3.5 mm air lines tested in this experiment are "noninsertable" and have male connectors on both ends. A true thru connection is not

possible with these lines. The 3.5 mm air lines are designed to be used as LRL (line-reflect-line) calibration standards. The propagation constant γ for the 3.5 mm air lines was measured using the LRL calibration technique as outlined in [3].

Various methods have been proposed for measuring the capacitance per unit length, C [4]. In this experiment, C was determined by measuring the air lines at 1 kHz on a capacitance bridge. As discussed in [1], C is nearly independent of frequency and metal conductivity so that measurements made at 1 kHz are applicable at microwave frequencies with negligible error. A frequency of 1 kHz was chosen because that is where capacitance measurements of high accuracy are possible.

First the total capacitance C_{t1} of the test ports and capacitance termination was measured without the air line. Then the air line was inserted and the total capacitance C_{t2} measured. Thus $C = (C_{t2} - C_{t1})/D$. This method essentially removes the capacitance of the test ports and termination from the measurements. It should be noted that accurate measurements of D are not required to determine Z_o since γD and CD are the quantities measured, and D is eliminated when the ratio $(\gamma D)/(CD)$ is calculated.

The measurements of C for the 3.5 mm air lines were complicated by their noninsertability. The capacitance of the test ports and termination cannot be "zeroed" since a thru connection is not possible. The capacitances for these lines were determined by measuring their capacitance relative to that of a short reference line whose capacitance was accurately known.

B. Dimensional Method

Various approximations have been used to calculate Z_o of a coaxial air line from dimensional measurements [5]-[7]. One of the more complete definition of Z_o is that given by Daywitt [8]. His derivation leads to the following approximation.

$$Z_o = Z_{oo}(1 - F_3), \qquad (3)$$

where

$$F_3 = (j - 1)\frac{\delta}{b}F_4, \qquad (4)$$

and where

$$F_4 = \frac{(1 + b/a)}{4\ln(b/a)}, \qquad (5)$$

and where

$a =$	radius of inner conductor,
$b =$	radius of outer conductor,
$\delta =$	skin depth $= 1/\sqrt{\pi f \mu \sigma}$,
$f =$	frequency, Hz
$\mu =$	permeability of free space $= 4\pi 10^{-7} H/m$,
$\sigma =$	conductivity of metals $\approx 1.3x107$ S/m for BeCu.

The quantity Z_{oo} is the characteristic impedance of a lossless line given by

$$Z_{oo} = \frac{\sqrt{\mu/\epsilon}}{2\pi}\ln(b/a), \qquad (6)$$

where

$\epsilon =$	dielectric constant $= \epsilon_o \epsilon_r$,
$\epsilon_o =$	dielectric constant in vacuum $= 8.8542 \times 10^{12}$ F/m,
$\epsilon_r =$	Relative dielectric constant,
$=$	1.000649 for air 23C, 50% rel. humidity, sea level,
$=$	1.000535 for air 23C, 50% rel. humidity, Boulder, CO.

The capacitance per unit length was calculated from dimensional measurements using

$$C = \frac{2\pi\epsilon_r\epsilon_o}{\ln b/a}. \qquad (7)$$

III. ACCURACY OF GAMMA METHOD

The propagation constant γ for an air line can be expressed as

$$\gamma = \frac{-\ln|S_{12}| - j\Psi_{12}}{D}, \qquad (8)$$

where $|S_{12}|$ and Ψ_{12} are the magnitude and phase respectively of the S-parameter S_{12}, and D is the length of the air line. Substituting (8) in (2) results in

$$Z_o = Z_r + jZ_i = -\frac{\Psi_{12}}{\omega CD} + \frac{j\ln|S_{12}|}{\omega CD}. \qquad (9)$$

For air lines, $|Z_o|$ is dominated by Z_r while ARG(Z_o) is dominated by Z_i.

As can be seen, the accuracy of the gamma method depends on the accuracy of the measurements of $|S_{12}|$, Ψ_{12}, and C. Note that Z_r depends on the measurements of Ψ_{12} while Z_i depends on $|S_{12}|$. At frequencies below 1 GHz, Ψ_{12} of air lines can often be measured more accurately than $|S_{12}|$. Air lines have low losses at low frequencies, and imperfections due to connectors can dominate the measurement of $|S_{12}|$.

Figs. 1 and 2 were prepared to show the uncertainty in measuring Z_o, assuming measurement errors of
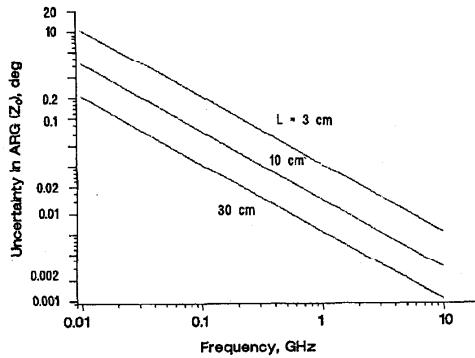
$$\Delta|S_{12}| = 0.01, \quad \text{db}$$

$$\Delta\Psi_{12} = 0.0005 + 0.0005 * FGHz, \quad \text{rad}$$

and

$$\Delta C = 0.0002 \text{ pF/cm}.$$

These assumed errors are typical of the measurement errors encountered during these tests. The figures shows results for air lines 3, 10, and 30 cm long. Note that the uncertainty in measuring $|Z_o|$ and ARG(Z_o) is inversely proportional to the length of the line. Thus the uncertainty in measuring a 3 cm air line is 10 times greater than that for a 30 cm air line.
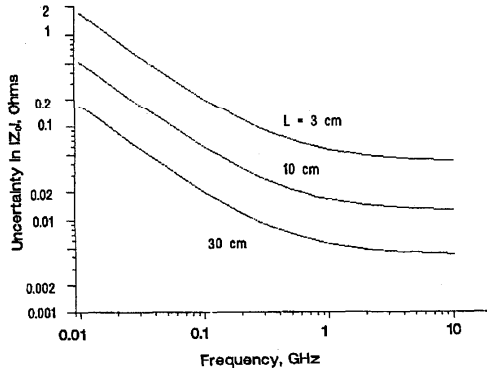
Fig. 1.   Uncertainty in measuring $|Z_o|$ using the gamma method.

Fig. 2.   Uncertainty in measuring phase of Z_o using the gamma method.
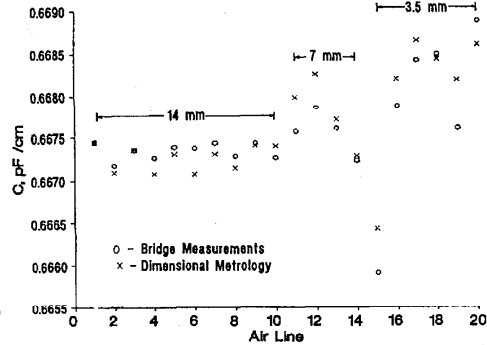

Fig. 3.   Comparison of capacitance per unit length from 1 kHz bridge measurements with calculations from dimensional measurements.

TABLE I
SUMMARY OF MEASUREMENTS OF C. VALUES OF ΔC ARE THE MAXIMUM DIFFERENCES THAT WERE OBSERVED FOR EACH OF THE LINE SIZES

b	$\Delta C, pF/cm$	Δa	Δb
14 mm	0.0003	2.3 μm (90 μin)	5.3 μm (207 μin)
7 mm	0.0004	1.5 μm (60 μin)	3.5 μm (138 μin)
3.5 mm	0.0006	1.1 μm (45 μin)	2.6 μm (103 μin)

If ΔC is due to an error Δa in the measurement of the center conductor diameter a, then from (7),

$$\Delta C = \frac{C\Delta a}{a\ln(b/a)} = \frac{0.8\Delta a}{a}, \quad \frac{pF}{cm}. \tag{10}$$

Similarly, if ΔC is due to an error Δb in the measurement of outer conductor diameter b, then

$$\Delta C = -\frac{C\Delta b}{b\ln(b/a)} = -\frac{0.8\Delta b}{b}, \quad \frac{pF}{cm}. \tag{11}$$

For 50 Ohm air lines, b/a = 2.3028. Values for Δa and Δb for each ΔC are shown in table 1 for each of the line sizes.

Measurements of Z_o for three different 14 mm air lines are shown in Figs. 4–9. Figs. 4–6 show $|Z_o|$, while Figs. 7–9 show the phase (ARG(Z_o)) . The lines are 30, 15, and 5 cm long. Results are shown for both the gamma method and the dimensional method. As predicted in the accuracy analysis, the measurement errors for the gamma method are inversely proportional to the length of the line. The measurement errors for the 5 cm line are roughly six times greater than for the 30 cm line. The perturbation that occurs in Fig. 6 at 4 GHz is due to the rf connectors. The perturbation changes with reconnection of the air line and can be eliminated in some connections. A perturbation of the magnitude shown can be attributed to a measurement error of $\Delta|S_{12}|$ =0.015 dB, and $\Delta\Psi_{12}$ =0.10 deg.

As noted previously, Z_r can often be measured moreaccurately than Z_i at frequencies below 1 GHz. Fortunately, independent measurements of Z_r and Z_i are not necessary since one can be determined from the other. As can be seen
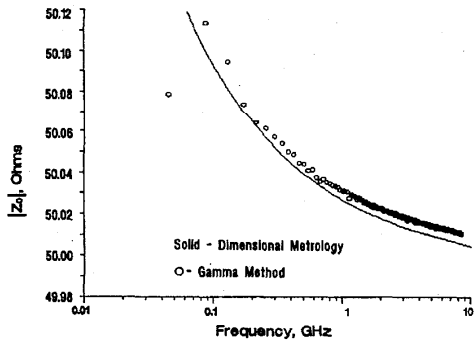
IV. EXPERIMENTAL RESULTS

The results of the measurements of C on the 1 kHz capacitance bridge are shown in Fig. 3. This figure shows both the measured values and the values calculated from dimensional measurements. Air lines numbered 1 to 10 are 14 mm, 11 to 14 are 7 mm, and 15 to 20 are 3.5 mm. The maximum difference between the measured and calculated values, ΔC, is given in Table I for each of the line sizes.

Generally, there is good agreement between the measurements of C on the low-frequency bridge and dimensional metrology. The quality and repeatability of the test ports and connectors is a major factor in the accuracy of the bridge measurements. The capacitance for some air lines would change significantly when they were disconnected and reconnected. The capacitance was also sensitive to the relative alignment of the test port and termination, which is probably indicative of the eccentricity in the connectors on these devices.

Fig. 4. $|Z_o|$ for a 14 mm air line, 30 cm long.

Fig. 5. $|Z_o|$ for a 14 mm air line, 15 cm long.

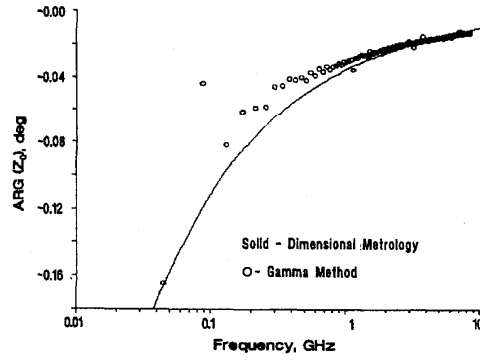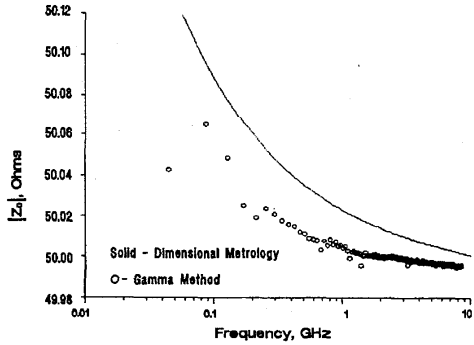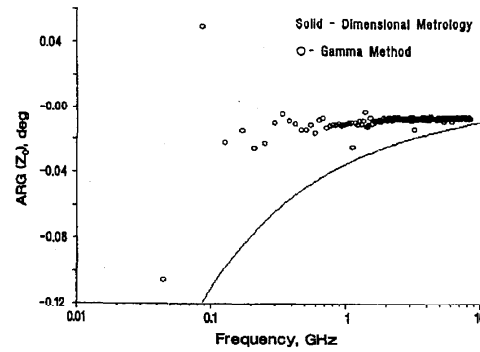Fig. 6. $|Z_o|$ for a 14 mm air line, 5 cm long.

Fig. 7. Phase of Z_o for a 14 mm air line, 30 cm long.

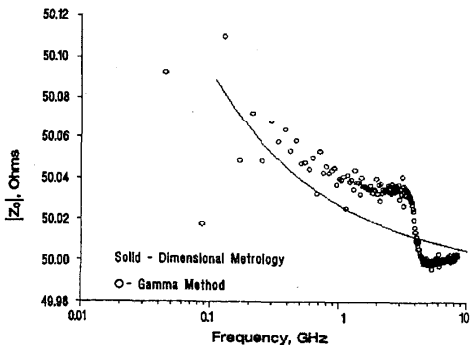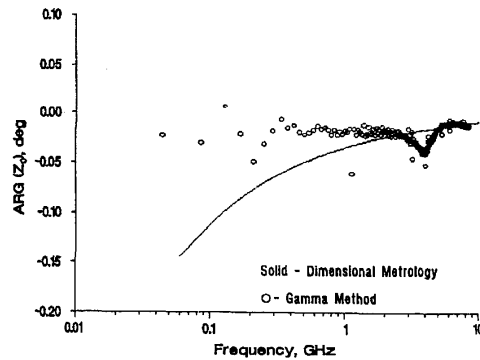Fig. 8. Phase of Z_o for a 14 mm air line, 15 cm long.

Fig. 9. Phase of Z_o for a 14 mm air line, 5 cm long.

from (3) and (4),

$$Z_o = Z_r + jZ_i \approx Z_{oo} + Z_{oo}(1-j)\frac{\delta}{b}F_4. \qquad (12)$$

Z_{oo} can be determined from the high-frequency measurements of Z_r since Z_r approaches Z_{oo} as the frequency increases.

Thus Z_i can be determined from measurements of Z_r using

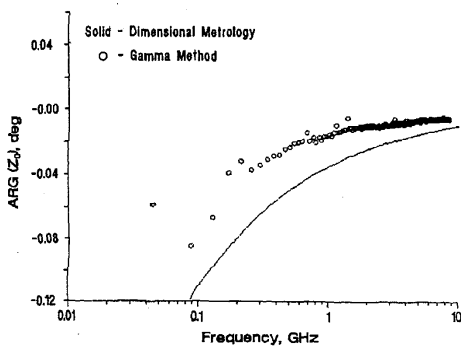$$Z_i = -Z_{oo}\delta\frac{F_4}{b} = Z_{oo} - Z_r. \qquad (13)$$

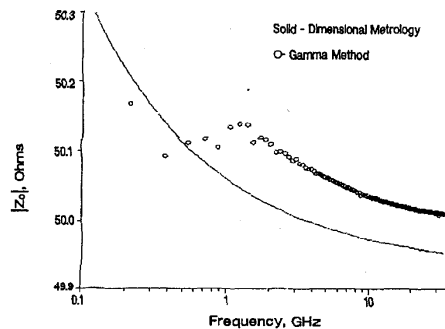Fig. 10. Phase of Z_o for a 14 mm air line, 15 cm long. With the gamma method. Z_i is calculated from Z_r.
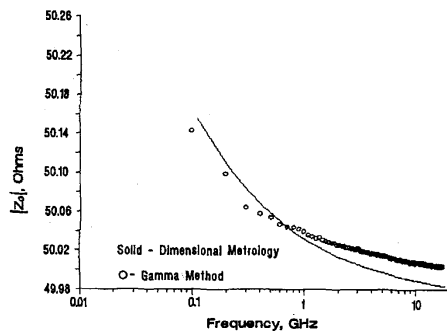

Fig. 11. $|Z_o|$ for a 7 mm air line, 15 cm long.

Fig. 12. $|Z_o|$ for a 3.5 mm air line, 12 cm long.

TABLE II
MAXIMUM DIFFERENCE IN $|Z_o|$ BETWEEN THE
GAMMA METHOD AND THE DIMENSIONAL METHOD

Air Line	Length, cm	$\Delta|Z_o|$, Ohms
14 mm	30	0.005
14 mm	15	0.03
14 mm	5	0.03
7 mm	15	0.03
3.5 mm	12	0.10

Fig. 10 shows the phase of Z_o when Z_i is computed from measurements of Z_r. Comparing Figs. 8 and 10 shows better agreement between the methods when Z_i is computed from Z_r.

Measurements of $|Z_o|$ for 7 mm and 3.5 mm air lines are shown in Figs. 11 and 12, respectively. The 7 mm air line is 15 cm long, and the 3.5 mm air line is 12 cm long. As noted, the 3.5 mm air line was measured using the LRL technique. With the LRL measurements a 3 cm line was used for the short line. Thus the effective measurement length of the 12 cm line is 9 cm.

A summary of the differences between the two methods is shown in Table II. The values shown are the maximum differences in $|Z_o|$ for frequencies greater than 0.1 GHz. Measurements at frequencies below 0.1 GHz were very noisy and contain little information.

V. CONCLUSION

The gamma method provides a means for electrically measuring the characteristic impedance of a coaxial air line independent of dimensional measurements. With the gamma method, the characteristic impedance is determined from measurements of the propagation constant, γ, and the capacitance per unit length, C. In these experiments, γ was measured on a commercial network analyzer, and C was measured at 1 kHz on a capacitance bridge. One of the advantages of the gamma method is that it measures both the air lines and test ports electrically. Problems with the test ports and connectors can be seen with this method. The disadvantage of the gamma method is that it is accurate only for long air lines where the phase shift and loss are sufficient for accurate measurements. Short air lines are still best characterized by dimensional measurements.

The accuracy of the gamma method is limited by imperfections in the test ports and connectors. Test ports often have dimensional tolerances that are significantly worse than those of air lines. The quality of the thru connection is important with the gamma method, and test port imperfections can introduce significant errors in that connection. Test port and connector errors are particularly evident in the measurement of lines smaller than 5 cm.

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