The Six-Port Reflectometer: An Alternative Network Analyzer

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Abstract—**Although the six-port measurement technique is rapidly gaining the attention of the microwave community, the theoretical development, to date, yields but limited practical insight into its operation.**

Following a brief review of the prior art, such that the six-port techniques may be placed in proper perspective, this paper presents an alternative introduction leading to much better insight, and to design criteria for a six-port circuit which optimally exploits the concept.

I. Background

Although the same basic circuit description is applicable in both cases, microwave measurement techniques represent an almost completely separate discipline as compared with measurements at lower frequencies. Not only is the phenomenon of interest usually enclosed within a conducting surface, which impedes the connection of measuring instruments, but of more fundamental concern, it is generally impossible to probe or sample the fields without significantly altering them.

A typical problem is illustrated in Fig. 1. Here the electromagnetic fields, at an arbitrarily chosen terminal plane in the uniform waveguide, are determined by the waveguide geometry and by the *complex* amplitudes b_2,a_2 of the waves proceeding to the right and left, respectively. Frequently, there is no interest in the absolute phase of a_2 or b_2 but only in their phase difference, and since the geometry is fixed, a complete description at the terminal surface requires only three independent parameters. These may be conveniently expressed as $|b_2|^2$ and Γ_l, where $\Gamma_l = a_2/b_2$. (The more general case where the absolute phase of a_2 or b_2 is required will not be explicitly considered.) These parameters, if they do not include the measurand of interest, usually play a major role in its determination, e.g., attenuation measurement.

Unfortunately, as a rule, one cannot make a direct measurement of a_2,b_2 as might be done at lower frequencies. Although conceptually straightforward, the introduction of a probe at the terminal plane destroys the postulated waveguide uniformity to which microwave circuit theory owes its existence. Ordinarily, one is prepared to go to great lengths to preserve this uniformity; this provides much of the motivation for the development and use of precision connectors. In particular, instead of using probes or other sampling means at the terminal plane, these are only permitted at remote positions as indicated in Fig. 1.

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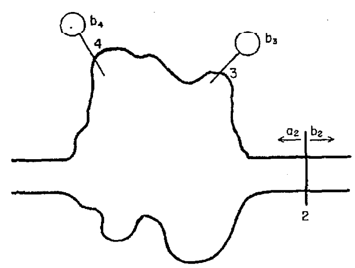

Fig. 1. A typical problem in microwave measurements.

Assuming that the intervening structure is linear, but otherwise arbitrary, it has been shown [1] that the probe response (at position 3) is given by

$$b_3 = Aa_2 + Bb_2. \tag{1}$$

Here b_3 is some linear combination of the complex electric and magnetic field amplitudes; A,B are complex constants (whose values are determined primarily by the intervening geometry) and a_2,b_2 are as previously defined. Obviously, at position 4 one has

$$b_4 = Ca_2 + Db_2 \tag{2}$$

where C,D are additional complex constants.

Ordinarily, the role of the metrologist includes not only observations of b_3,b_4, but the determination of $A \cdots D$ as well. Given this information, a_2,b_2 may be found by solving a linear system of simultaneous equations. The metrologist's problems, however, do not end here. Although in some cases the provisions for this remote sampling may be built into a microwave system, more often this is not the case. Take for example the simple problem of measuring the power being delivered to an antenna. In general this requires a measurement of its complex impedance and the Thevenin (or other) equivalent circuit of the source to which it is connected. Thus the metrologist has to measure *five* parameters to get at the *one* of interest. Although certain approximate methods for this problem are in widespread use, there is increasing pressure to eliminate the (mismatch) errors which are thus introduced. Apart from "power equation" methods [2], [3], this generally calls for the indicated approach.

II. Review of Prior Art

Although conceptually straightforward, the application of (1) and (2) requires that the detectors provide *phase* as well as amplitude response. Because of the overall complexity associated with phase detection at microwave frequencies, much of the prior art was built around detectors which yielded only amplitude or power response.

Taking the absolute value of (1) leads to

$$P_3 = |b_3|^2 = |Aa_2 + Bb_2|^2 \tag{3}$$

where P_3 is the power response at port 3, while at port 4

$$P_4 = |Ca_2 + Db_2|^2. \tag{4}$$

It will prove useful to briefly review the earlier applications of these equations in the microwave art.

The familiar slotted line, for example, is a device which is so constructed that (ideally) $|A| = |B|$ while the phase between A and B is adjustable (by means of the probe position). By taking the ratio of maximum to minimum response, the parameters A,B are eliminated, and $|\Gamma_l|$ is obtained. The phase of Γ_l can also be obtained by well-known means; however, the slotted line technique does not lend itself to a determination of $|b_2|^2$, and thus power.

The reflectometer, on the other hand, requires two detectors (P_3,P_4) and ideally, $B = C = 0$. In this case $P_3 = |A|^2|a_2|^2$, $P_4 = |D|^2|b_2|^2$ and if $|A|^2$ and $|D|^2$ are known, one can measure $|b_2|^2$, $|a_2|^2$ and thus power; or if only $|A/D|^2$ is known, this still permits one to measure $|\Gamma_l|$. In its elementary form, this scheme does not provide the phase of Γ_l.

In order to obtain the phase, additional detectors can be added. Measurement schemes of this type have been described by Samuel [4], Cohn [5], and others. As before, however, certain idealized assumptions are made as to the nature of the probe response.

All of these measurement methods suffer from a common problem, namely that *the accuracy obtained is dependent upon how well certain design objectives for a particular item of hardware are realized.* As a means of circumventing this limitation, the tuned reflectometer was introduced. This permitted the metrologist to adjust the system parameters such that the idealized response was more nearly achieved. Over a period of perhaps a decade, this became a highly developed art. Unfortunately, however, these schemes proved both frequency sensitive and time consuming.

With the increasing pressure of broad-band requirements and the advent of digital techniques, the well-known automatic network analyzer (ANA) emerged. At the same time, *a major shift in measurement strategy was introduced.* Instead of attempting to construct an ideal microwave circuit, e.g., reflectometer, its imperfections were explicitly recognized, accounted for, and finally eliminated from the measurement results. Today, quite generally, the trend is away from stringent specifications and in the direction of more intelligent use of imperfect hardware.

In its current form, the ANA is based on (1) and (2). As a preliminary step, the system is "calibrated" by means of known standards. This leads to values for $A \cdots D$. (Actually only the ratio of three of these to the fourth is required in most applications.) Following this, the system may be solved for a_2,b_2 in terms of b_3,b_4. Because three complex parameters must be determined, the calibration aspect is more complicated than in the earlier systems where only one or two scalar parameters were required. This increased complexity, however, is more than compensated for by the other time-saving features which the automation provides. The existing ANA's are a complicated piece of equipment, and a fair amount of this complexity is associated with the requirement imposed by (1) and (2), namely that the detectors (b_3,b_4) provide phase as well as amplitude.

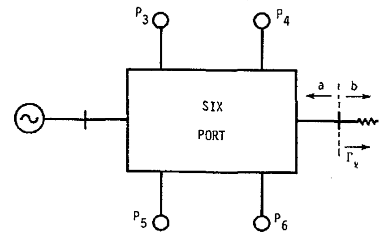

Fig. 2. Six port for measuring complex microwave parameters.

III. The Six-Port Reflectometer

The six-port reflectometer provides an alternative method of implementing the ANA. In common with the existing designs, hardware imperfections are taken care of via the software. The distinguishing feature is that the operation is based on equations of the form of (3) rather than (1). The requirement for phase information is thus avoided, and simple amplitude or power detectors may be used rather than complex heterodyne detection schemes. As a further consequence, practical experience, to date at least, indicates that the frequency sensitivity has been reduced so that a phase locked source is no longer required (although it may still be convenient in many cases). Because of these simplifications, the method is easily extended to millimeter frequencies.

In addition to (3) and (4), the response of the six-port reflectometer (Fig. 2) is contained in two additional power meter readings:

$$P_5 = |Ea_2 + Fb_2|^2 \tag{5}$$

$$P_6 = |Ga_2 + Hb_2|^2 \tag{6}$$

where $E \cdots H$ are additional complex constants. For the purpose of this paper, the parameters $A \cdots H$ are assumed to be known. Techniques for their measurement will be described in a subsequent paper.

In earlier papers [6], [7] it was shown that one solution to (3)–(6) is given by

$$|b_2|^2 = \sum_{i=3}^{6} \beta_i P_i \tag{7}$$

and

$$\Gamma_l = \frac{\sum_{i=3}^{6} (C_i + jS_i)P_i}{\sum_{i=3}^{6} \beta_i P_i} \tag{8}$$

where C_i, S_i, and β_i are real and functions of $A \cdots H$. In arriving at this solution, however, the fact that the system is overdetermined (i.e., only three detectors are required) was ignored. Moreover, this approach yields but little, if any, insight as to how to choose $A \cdots H$ so as to best exploit the method. These shortcomings are corrected in what follows.

Apart from the design criteria which may emerge from a study of (3)–(6), there are the additional practical requirements of correcting for power instability in the signal source and ensuring that the power levels at the several detectors and output port are maintained at some optimum value as the frequency is varied. Ordinarily, this calls for a feedback loop and, unless otherwise provided for, an additional directional coupler or other means to sample and measure the incident wave amplitude $|b_2|$. The measurement of $|b_2|$, however, represents a determination of one of the measurands of interest. Thus there is a double role served by designing the six port in such a way that the response of one of the power meters is proportional to $|b_2|^2$. In order to provide continuity with the terminology in prior papers, the port chosen for this role is number 4. Referring to (4), the first design objective is that $C = 0$, and to the extent that this condition is realized (4) becomes

$$P_4 = |D|^2 |b_2|^2. \tag{9}$$

In order to explicitly display the measurands of interest, (3), (5), and (6) may be written as

$$P_3 = |A|^2 |b_2|^2 |\Gamma_l - q_3|^2 \tag{10}$$

$$P_5 = |E|^2 |b_2|^2 |\Gamma_l - q_5|^2 \tag{11}$$

$$P_6 = |G|^2 |b_2|^2 |\Gamma_l - q_6|^2 \tag{12}$$

where $q_3 = -B/A$, $q_5 = -F/E$, and $q_6 = -H/G$.

It is possible to let $|b_2|$ and Γ_l represent a point in three-dimensional space and to discuss the problem in terms of three-dimensional geometry. A more convenient approach, however, is to first eliminate $|b_2|^2$ from (10), (11), and (12) by means of (9). This leads to a problem in two dimensions. Although (9) is only an approximation, it will prove convenient to initially treat it as exact and then consider the general case.

Elimination of $|b_2|^2$ between (9) and (10), for example, leads to

$$|\Gamma_l - q_3|^2 = \left|\frac{D}{A}\right|^2 \cdot \frac{P_3}{P_4}. \tag{13}$$

Let Fig. 3 represent the Γ_l plane. Ordinarily, the terminations to be measured are passive ($|\Gamma_l| \leq 1$) so that Γ_l falls within the unit circle as shown. For reasons which will emerge, it is convenient to assume initially that q_3 lies outside this circle. Given the measurement results P_3, P_4, and assuming q_3 and $|D/A|^2$ are known, the locus of possible values for Γ_l is a circle with center at q_3 and whose radius, $|\Gamma_l - q_3|$, may be determined from (13).

In the same way (9) and (11) may be combined, and the radius of another circle, which contains Γ_l, with center at q_5 determined. The situation is now as shown in Fig. 4. Here Γ_l

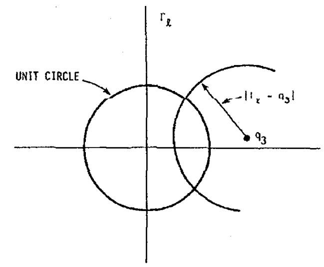

Fig. 3. Locus of possible values for Γ_l determined by P_3 and P_4.

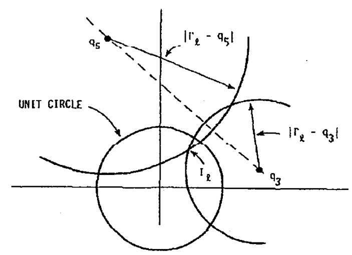

Fig. 4. Determination of Γ_l from the intersection of two circles.

is determined by the intersection of the two circles. Two circles, however, intersect in a pair of points. In this example the second point falls outside the unit circle, and one is able to choose between the two solutions on the basis $|\Gamma_l| \leq 1$.

Thus far, no use has been made of P_6, and the system may be considered a five port rather than six port. Before introducing P_6, some additional observations on the five-port mode are of interest. As already noted, the five-port mode leads to a pair of values for Γ_l. Provided, however, that a straight line between q_3 and q_5 does not intersect the unit circle, one is assured that one of these roots will fall outside of it and (assuming a passive termination) may be rejected on this basis.

By further inspection of Fig. 4, one notes that the angle at which the circles intersect is rather small and it is easily recognized that the position of Γ_l, in a direction perpendicular to the line between q_3 and q_5, has a high sensitivity to errors in $|\Gamma_l - q_3|$ or $|\Gamma_l - q_5|$. In the parallel direction, the sensitivity is appreciably less. Over the range of possible choices for Γ_l, and in particular if Γ_l moves around the perimeter of the unit circle, one can expect a considerable variation in these sensitivities or expected errors in a practical measurement system.

At first glance one might be tempted to relieve this problem by increasing the distance from q_3 and q_5 to the origin. For example, if $q_3 = 10$ and $q_5 = j10$, the intersection of the respective circles will be nearly orthogonal over the entire unit circle. Unfortunately, however, this superficial improvement is more than offset by other considerations. In the example just given, a little further study would show that a 1-percent error in measuring $|\Gamma_l - q_3|$ or $|\Gamma_l - q_5|$ would translate respectively into a nominal uncertainty of 10 percent in the real and imaginary parts of Γ_l.

On the basis of this discussion it should be apparent that the choice of optimum values for q_3 and q_5 represents a compromise between a number of conflicting requirements. How one chooses to resolve this conflict will depend in part, for example, upon how much variation in accuracy at the perimeter of the unit circle one is prepared to accept in return for improved accuracy at its center.

Although the five-port measurement concept is technically sound, the prime purpose for this discussion has been to prepare one to appreciate further the benefits of a six-port versus five-port approach. Because these improvements are substantial, the future for the five port appears limited. For this reason the five port will not be considered in further detail in this paper.

To continue, q_6 is chosen as shown in Fig. 5 and $|\Gamma_l - q_6|$ is determined from (12) and (9). This provides a third circle upon which Γ_l must lie and which (ideally) must pass through the intersection of the other two circles as shown in Fig. 5. In practice, because of measurement errors, the three circles will not intersect in a point, and some sort of statistical weighting is called for. Although it is not within the scope of this paper to consider this aspect in detail, it is intuitively obvious that this additional detector has substantially enhanced the accuracy with which Γ_l may be determined. In particular, the position of Γ_l in the direction orthogonal to the line between q_3 and q_5, and which was quite sensitive to errors in P_3 and P_5, may now be inferred primarily from $|\Gamma_l - q_6|$ and with less sensitivity to error. Moreover, the double root ambiguity has also been resolved; no longer is it required that the line connecting q_3 and q_5 lie outside the unit circle.

Following this general approach, the system may be expanded to seven or more ports. With the possible exception of a seven port, however, the accuracy improvement does not ordinarily warrant the additional complexity.

In the discussion thus far, it has been assumed that (9) was satisfied; at best this is only approximately true. Unfortunately, a complete discussion of the more general case is lengthy, and many of the conclusions will be stated without detailed proof. In order to generalize the approach, it is convenient to ignore P_4 and begin by eliminating $|b_2|^2$ between (10) and (12). This leads to

$$\frac{|\Gamma_l - q_3|^2}{|\Gamma_l - q_6|^2} = \left|\frac{G}{A}\right|^2 \frac{P_3}{P_6}. \tag{14}$$

If one expands this result, it can be shown that the locus of possible values for Γ_l is again a circle, with center (somewhere) on the line through q_3 and q_6. If the ratio P_3/P_6 is permitted to take on different values, a family of circles, each corresponding to a different value of P_3/P_6, is generated as shown in Fig. 6. It is of interest and easily shown that this family of circles is that which is also used to illustrate the surfaces of constant potential associated with a parallel wire transmission line, and where q_3,q_6 correspond to the positions of the conductors. As already noted, for a given value of P_3/P_6, the locus of Γ_l is a circle, but unlike the previously described ideal case, the position of the center, as well as the radius, is a function of P_3/P_6. In a similar manner, (11) and

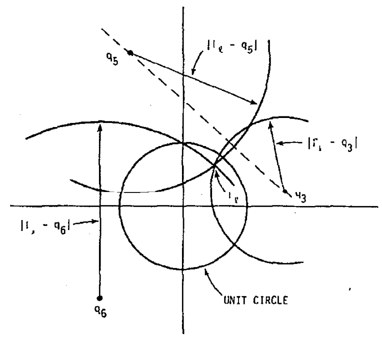

Fig. 5. An improved determination of Γ_l from the intersection of three circles.

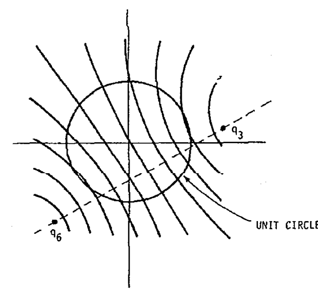

Fig. 6. Generalization of problem to the case where equation (9) is not satisfied.

(12) may be combined, leading to another circle, this time with the center somewhere on the line through q_5 and q_6. As before, Γ_l is determined by the intersection of two circles, and two possible values of Γ_l are obtained. For large values of $|q_6|$, the role played by P_6 approaches that previously filled by P_4 and becomes identical to it as $|q_6| \to \infty$. Thus the primary correction to the prior description, which was based on (9), is to recognize that the respective circles are not centered at q_3 or q_5 although this is usually a good approximation.

Finally, one may also combine (10) and (11). This also leads to a circle, this time with center on the line which connects q_3 and q_5. It can be shown that this center also lies on the line which connects the centers of the two previously determined circles and passes through their points of intersection. Thus there is no additional information to be gained by this exercise.

Because P_4 has been ignored, it will be recognized that this discussion pertains to a five port. Although not its primary objective, the discussion has also provided insight into possible designs if, for some reason, leveling is not required. The extension to a six port follows along the lines already presented.

IV. Design Criteria

As noted in an earlier paragraph, and referring again to (4), the first design objective ordinarily is that $C = 0$. This leads to (9). Although nothing has been said, thus far, about the choice of $|D|$, $|A|$, $|E|$, and $|G|$, it is immediately evident from inspection of (9)–(12) that these are scale factors, which for a given signal at the output port, determine the power levels at the several power meters. Ordinarily, these parameters are chosen such that these levels are compatible with the power meter characteristics.

The major design question centers around the choice of q_3, q_5, and q_6. One representative set of values is shown in Fig. 5. However, it is appropriate to ask if a better choice would be to place one of the q's, say q_3, at the center of the unit circle. If this is done, one has one response (P_4) which measures the incident wave ($|b_2|$) while P_3 now measures the reflected wave ($|a_2|$). In this case the six port incorporates the reflectometer, a device which has played a substantial role in the prior art. There are several considerations, however, which argue against this choice for q_3. Assuming one could obtain the condition $q_3 = 0$, the prospect of achieving a direct measure of the reflection coefficient magnitude is indeed attractive. In actual fact, in the current state of the art, and even with this as a design goal, the expected deviations of q_3 from zero are such as to largely negate the potential advantages. A more serious objection arises from dynamic range versus measurement precision considerations. This point is perhaps best illustrated by a specific example.

In Fig. 5 let q_3 be moved to the center of the diagram, let $q_5 = 2$, and $q_6 = j2$. Bolometric-type power meters will be assumed, for which typical performance specifications include upper power limit—10 mW and error—0.1 percent $\pm$ 1 μW. Next, the values of $|A|$, $|E|$, $|G|$ are chosen such that the 10-mW limit may be approached (but not exceeded) for all possible values of Γ_l, where $|\Gamma_l| \leq 1$.

If one now wishes to measure a termination for which $|\Gamma_l| \sim 0.01$, P_3 will be approximately 1 μW, thus the signal-to-noise ratio for this detector has dropped to unity. By contrast, P_5 and P_6 will be operating at approximately 5 mW, and the 0.1 percent will be the dominating error term. Since this applies to power, the error in $|\Gamma_l - q_5|$ and $|\Gamma_l - q_6|$ will be half of this. On the other hand, the nominal value of $|\Gamma_l - q_5|$ or $|\Gamma_l - q_6|$ in this example is 2, so that the uncertainty in the real and imaginary parts of a value of Γ_l in the neighborhood of the origin is ± 0.001. Loosely speaking, this represents a 10-percent error for $|\Gamma_l| \sim 0.01$. The interesting and unexpected conclusion is that if one requires operation over the entire range $|\Gamma_l| \leq 1$, a point in the neighborhood of the origin can be located more precisely in terms of its distances from points which are somewhat removed than from a point in its immediate neighborhood.[1] Returning to the example just given, the response of P_3 contributes little or nothing to the determination of Γ_l when $|\Gamma_l|$ is small, and it appears that a better choice of q_3 would be as shown in Fig. 5. Although the foregoing arguments do not necessarily hold for all choices of power meters, they do appear valid for the immediate candidates which include the bolometric and diode types.

Having disposed of the question of placing one of the q's at the center of the unit circle, it now appears, from symmetry considerations, that q_3, q_5, and q_6 should be located at the vertices of an equilateral triangle whose center is at the origin. (A rigorous proof of this conclusion, unfortunately, is difficult and has not been undertaken at this writing.) If one accepts this proposition, this calls for $|q_6| = |q_5| = |q_3|$, while the arguments differ by $\pm 120°$. Thus the only remaining choice is the magnitude $|q_3|$ of these terms. An earlier paragraph has already commented on the errors which result from making this too large. Although the use of four detectors has eliminated the ambiguity problem, and thus permits one to choose $|q_3| < 1$, there are easily recognized similar problems if $|q_3|$ becomes too small. In particular, since Γ_l is determined from its distances from q_3, q_5, and q_6, it is evident that an ill-conditioned situation will result if these distances become large in comparison with the distances between q_3 and q_5, q_3, and q_6, or q_5 and q_6. On the basis of these last considerations, it appears that an optimum value for $|q_3|$ might be expected to lie in the range 0.5–1.5. However, for reasons which will be developed in a subsequent paper, the calibration techniques referred to earlier become poorly conditioned if $|q_3| \simeq 1$. Moreover, an experimental study with the aid of a computer shows a decrease in the measurement accuracy when $|\Gamma_l| \simeq |q_3|$. Since there is usually a substantial interest in values of Γ_l with a nominal magnitude of unity, there is a double reason for avoiding $|q_3| \simeq 1$. Apart from values close to unity, the other region of primary interest is $|\Gamma_l| \leq 0.3$. In order to provide the largest possible bandwidth, a fairly loose tolerance on the performance of the individual components from which the six port is constructed is desirable. This now reduces the choices for $|q_3|$ to values in the neighborhood of 0.5 or 1.5.

[1] For a further discussion of this point, see [8].

V. Summary

This paper provides an alternative theoretical introduction to the six-port measurement technique, which yields a far more ready physical insight into its operation than the earlier descriptions. As a consequence, it is now possible to design improved six-port circuits and apply them with greater confidence.

A companion paper [9] will describe the practical realization of a six-port circuit which approximates the design criteria developed in this paper.

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