

CHAPTER ONE: ARITHMETIC OPERATIONS

COURSE TERMINAL OBJECTIVE:

Given a calculator, apply the basic concepts and principles of arithmetic, algebra, trigonometry, and calculus to perform mathematical analyses and solve problems. Proficiency is demonstrated by achieving 70% or better on written examinations which sample lesson objectives.

LESSON TERMINAL OBJECTIVE:

Apply the basic concepts and principles of arithmetic, as outlined in this lesson. Proficiency is demonstrated by achieving 70% or better on a written examination which samples lesson objectives.

LESSON ENABLING OBJECTIVES:

- E01: Without the use of a calculator, add, subtract, multiply and divide rational numbers.
- E02: Express arithmetic complex fractions in simplest form.
- E03: Solve percentage problems.
- E04: Multiply and divide real numbers expressed as powers, expressing results as powers or as numbers free of exponents.
- E05: Given an arithmetic expression containing real numbers in scientific notation, determine the value of the expression in proper scientific notation.
- E06: Given an arithmetic expression involving the basic operations, parentheses, and powers, determine the value of the expression.

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1.1 BASIC ARITHMETIC

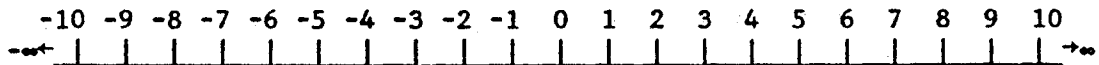
1.1.1 INTEGERS

In this section we define the integers and introduce some of their properties.

DEFINITIONS

- NATURAL NUMBERS** - The set of numbers used for counting: $[1, 2, 3, 4, \dots]$
- WHOLE NUMBERS** - A collection of numbers used in a counting system to represent quantity. The set consists of the natural numbers, and zero: $[0, 1, 2, 3, \dots]$
- INTEGERS** - The set of whole numbers and the negatives of those whole numbers: $[\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots]$
- NUMERAL** - A symbol used to denote a number. (the symbols in the sets above are examples of numerals)
- LITERAL NUMBER** - A number denoted by a letter

Integers can be represented by their location on a number line as follows:



Each numeral shown on the number line is a measure of the distance of that point on the line from zero. All numbers shown to the right of zero are positive integers; those to the left of zero are negative integers. Notice that the distance between any two consecutive integers is the same. Infinity (∞) is not used to denote a specific number; it indicates that the line is "endless".

A QUANTITY LESS THAN ZERO IS REPRESENTED BY A NEGATIVE (-) NUMBER.

A QUANTITY GREATER THAN ZERO IS REPRESENTED BY A POSITIVE (+) NUMBER.

A NUMBER WRITTEN WITHOUT A SIGN IS ASSUMED TO BE A POSITIVE NUMBER.

A NUMBER WITH ITS SIGN REPRESENTS THE ALGEBRAIC VALUE OF THE NUMBER.

An integer is considered to be greater than another (HAS A GREATER ALGEBRAIC VALUE) if it occurs to the right of the other on the number line. For example +3 is greater than +2. Note that for numbers less than zero, -2 is greater than -3.

1.1.1.1 ABSOLUTE VALUE

The quantity of units in a number, regardless of sign, is the absolute or numerical value of the number; its magnitude. Two numbers having the same absolute value but different signs are different numbers. THEY DIFFER IN ALGEBRAIC VALUE.

+3 IS A DIFFERENT NUMBER THAN -3

The absolute value of a number may be thought of as merely its distance from zero on the number line. When an absolute value is desired, the number is written between two parallel lines.

$$|+3| = 3$$

ABSOLUTE VALUE OF +3 = 3

$$|-3| = 3$$

ABSOLUTE VALUE OF -3 = 3

EXERCISES

A. Determine the absolute value of the following;

1) $|-4| = 4$

4) $|2| =$

2) $|+10| =$

5) $|-35| =$

3) $|-10| =$

6) $|236| =$

B. Which number in each pair has the larger absolute value?

1) $+36, +35$ ANSWER: $= +36$

2) $-36, +25$

3) $-21, -42$

4) $-9, +5$

5) $0, -20$

6) $0, +20$

C. Which number in each pair has the larger algebraic value?

1) $0, -20$

2) $0, +20$

3) $-5, -9$

4) $+10, +16$

5) $-8, +2$

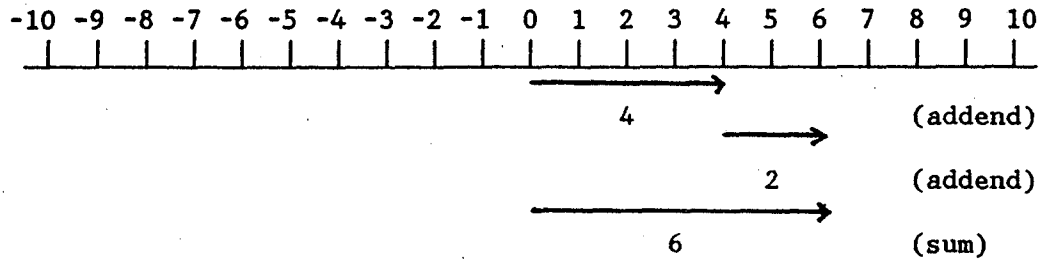
6) $+8, -2$

1.1.1.2 ADDITION OF SIGNED NUMBERS

THE SUM OF TWO ADDENDS (NUMBERS TO BE ADDED) HAVING LIKE SIGNS IS THE SUM OF THEIR ABSOLUTE VALUES PRECEDED BY THE COMMON SIGN.

The sum of 4 and 2 may be shown in the following ways:

EXAMPLE 1a



EXAMPLE 2a

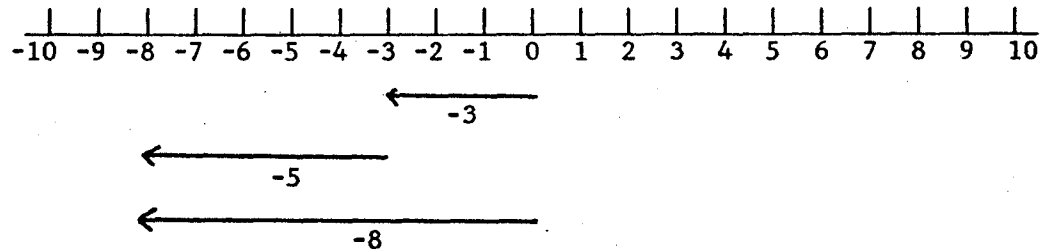
$$\begin{array}{r} +4 \\ +2 \\ \hline +6 \end{array}$$

EXAMPLE 3a

$$(4) + (2) = 6$$

The sum of -3 and -5 may be shown in the following ways:

EXAMPLE 1b



EXAMPLE 2b

$$\begin{array}{r} -3 \\ -5 \\ \hline -8 \end{array}$$

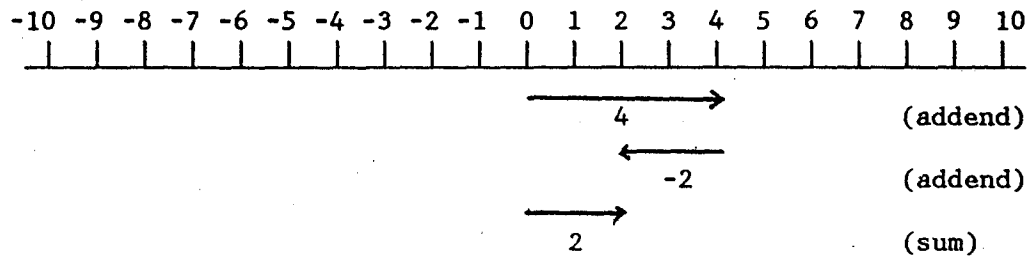
EXAMPLE 3b

$$(-3) + (-5) = -8$$

THE SUM OF TWO ADDENDS HAVING UNLIKE SIGNS IS THE DIFFERENCE OF THEIR ABSOLUTE VALUES PRECEDED BY THE SIGN OF THE NUMBER HAVING THE LARGEST ABSOLUTE VALUE.

The sum of 4 and -2 may be shown in the following ways;

EXAMPLE 4a



EXAMPLE 5a

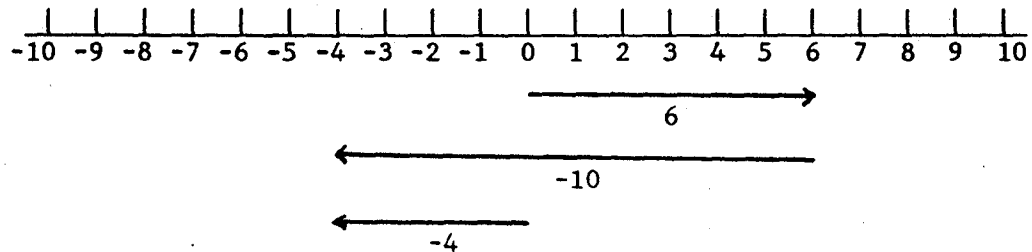
$$\begin{array}{r} +4 \\ -2 \\ \hline +2 \end{array}$$

EXAMPLE 6a

$$(4) + (-2) = 2$$

The sum of 6 and -10 may be shown in the following way:

EXAMPLE 4b



EXAMPLE 5b

$$\begin{array}{r} 6 \\ -10 \\ \hline -4 \end{array}$$

EXAMPLE 5c

$$6 + (-10) = -4$$

EXERCISES

Find the following sums:

- | | | | |
|-----|-----|------------|------------|
| 1) | +36 | <u>+27</u> | |
| 2) | +36 | <u>-27</u> | |
| 3) | -48 | <u>+24</u> | |
| 4) | -16 | <u>-14</u> | |
| 5) | -12 | <u>0</u> | |
| 6) | 116 | <u>125</u> | |
| 7) | -36 | <u>-85</u> | |
| 8) | +6 | <u>+5</u> | <u>-9</u> |
| 9) | -12 | <u>-10</u> | <u>+22</u> |
| | | <u>+11</u> | |
| 10) | +5 | <u>-10</u> | <u>+16</u> |
| | | | <u>-2</u> |

Find the following sums:

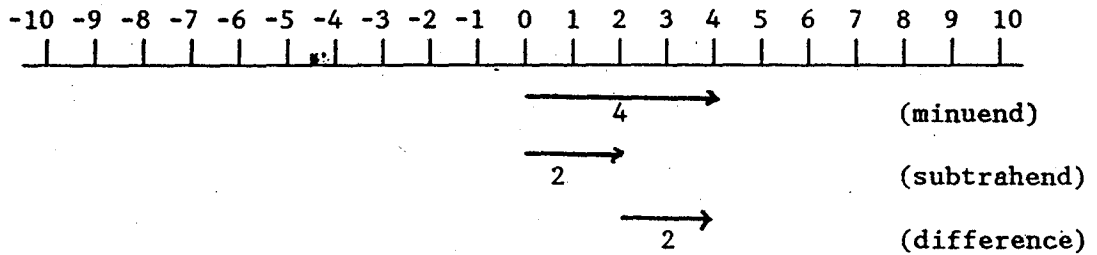
- 11) $(-16) + (14) =$
 12) $(42) + (-12) =$
 13) $(-10) + (-6) + (8) =$
 14) $(6) + (8) + (-4) =$
 15) $(-18) + (10) + (-6) + (10) =$

1.1.1.3 SUBTRACTION OF SIGNED NUMBERS

TO FIND THE DIFFERENCE OF TWO SIGNED NUMBERS, CHANGE THE SIGN OF THE SUBTRAHEND (NUMBER TO BE SUBTRACTED) AND ADD AS EXPLAINED IN SECTION 1.1.1.2.

The difference of 4 and 2 may be shown in the following ways;

EXAMPLE 7



EXAMPLE 8

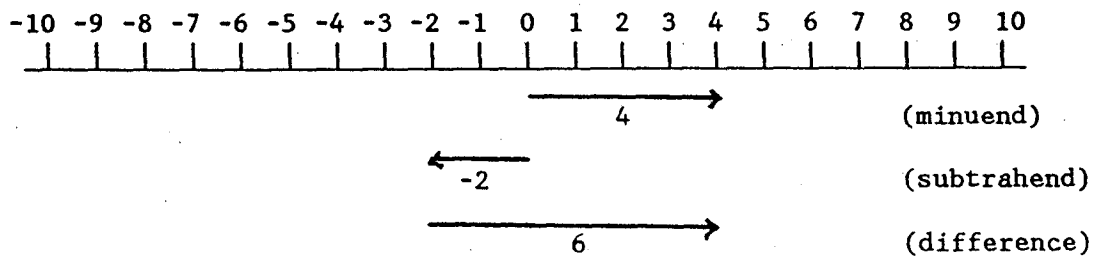
$$\begin{array}{r}
 +4 = +4 \\
 (-)+2 = -2 \\
 \quad +2
 \end{array}
 \leftarrow \text{(change sign of the subtrahend and add)}$$

EXAMPLE 9

$$(4) - (2) = (4) + (-2) = 2$$

The difference of 4 and -2 may be shown in the following ways;

EXAMPLE 10



EXAMPLE 11

$$\begin{array}{r} +4 = +4 \\ (-)-2 = +2 \\ \hline +6 \end{array} \leftarrow \begin{array}{l} \text{(change sign of the} \\ \text{subtrahend and add)} \end{array}$$

EXAMPLE 9

$$(4) - (-2) = (4) + (2) = 6$$

EXERCISES

Subtract

1) $\begin{array}{r} +100 \\ +100 \end{array}$

2) $\begin{array}{r} +45 \\ +15 \end{array}$

3) $\begin{array}{r} -45 \\ -15 \end{array}$

4) $\begin{array}{r} -20 \\ +16 \end{array}$

5) $\begin{array}{r} +125 \\ -136 \end{array}$

6) $\begin{array}{r} -30 \\ -42 \end{array}$

7) $(75) - (25) =$

8) $(75) - (-25) =$

9) $100 - 16 =$

10) $42 - (-12) =$

11) $(-68) - (-14) =$

12) $(-36) - (+9) =$

13) $(+8) - (+12) =$

14) $(-54) - (-60) =$

15) $96 - (-114) =$

1.1.1.4 MULTIPLICATION OF SIGNED NUMBERS

THE PRODUCT OF TWO NUMBERS OF THE SAME SIGN IS A POSITIVE NUMBER

EXAMPLE 13

$$(+4) \text{ times } (+5) = +20$$

$$(4) \quad \times \quad (5) = +20$$

$$4 \quad \bullet \quad 5 = +20$$

$$(4)(5) = +20$$

EXAMPLE 14

$$(-4) \text{ times } (-5) = +20$$

$$(-4) \quad \times \quad (-5) = +20$$

$$-4 \quad \bullet \quad -5 = +20$$

$$(-4)(-5) = +20$$

THE PRODUCT OF TWO NUMBERS HAVING UNLIKE SIGNS IS A NEGATIVE NUMBER.

EXAMPLE 15

$$(+4) \text{ times } (-5) = -20$$

$$(4) \quad \times \quad (-5) = -20$$

$$4 \quad \bullet \quad -5 = -20$$

$$(4)(-5) = -20$$

EXAMPLE 16

$$(-4) \text{ times } (+5) = -20$$

$$(-4) \quad \times \quad (5) = -20$$

$$-4 \quad \bullet \quad 5 = -20$$

$$(-4)(5) = -20$$

THE PRODUCT OF ANY NUMBER AND ZERO IS ZERO.

EXERCISES

Find the product of the following:

1) $(+10) \times (+6) =$

7) $(-12)(12) =$

2) $10 \bullet 4 =$

8) $(-16)(-8) =$

3) $(14)(3) =$

9) $(4)(-2)(3) =$

4) $(-6)(-2) =$

10) $(-6)(-2)(-3) =$

5) $(-75)(2) =$

11) $(-4)(0) =$

6) $(13)(-4) =$

12) $(2)(-6)(0) =$

1.1.1.5 DIVISION OF SIGNED NUMBERS

THE QUOTIENT OF TWO NUMBERS OF THE SAME SIGN IS A POSITIVE NUMBER.

EXAMPLE 17

$$18 \div 9 = 2$$

$$\frac{18}{9} = 2$$

$$9 \overline{)18} = 2$$

EXAMPLE 18

$$-18 \div -9 = +2$$

$$\frac{-18}{-9} = +2$$

$$-9 \overline{)-18} = +2$$

THE QUOTIENT OF TWO NUMBERS HAVING UNLIKE SIGNS IS A NEGATIVE NUMBER.

EXAMPLE 19

$$-18 \div 9 = -2$$

$$\frac{-18}{9} = -2$$

$$9 \overline{) -18} = -2$$

EXAMPLE 20

$$18 \div (-9) = -2$$

$$\frac{18}{-9} = -2$$

$$-9 \overline{) 18} = -2$$

WHEN ZERO IS DIVIDED BY ANY NON-ZERO NUMBER, THE QUOTIENT IS ZERO.
WHEN A SIGNED NUMBER IS DIVIDED BY ZERO, THE QUOTIENT IS INDETERMINATE.
TO DIVIDE BY ZERO IS UNDEFINED!

EXERCISES

1) $(-24) \div (12) =$

7) $\frac{+64}{-16} =$

2) $(42) \div (-3) =$

8) $\frac{0}{-12} =$

3) $(-36) \div (-4) =$

9) $\frac{-96}{-16} =$

4) $0 \div (-14) =$

10) $\frac{-161}{7} =$

5) $\frac{+75}{+15} =$

11) $\frac{651}{21} =$

6) $\frac{-28}{+4} =$

12) $\frac{12}{0} =$

1.1.1.6 PRACTICE PROBLEMS

Find the following sums:

1) $\begin{array}{r} -48 \\ +36 \\ \hline \end{array}$

4) $39 + 75 =$

2) $\begin{array}{r} -546 \\ -584 \\ \hline \end{array}$

5) $-16 - 24 + 10 =$

3) $\begin{array}{r} +742 \\ -805 \\ \hline \end{array}$

6) $100 - 84 + 0 - 16 =$

Find the following differences:

7) $(42) - (-6) =$

10) $0 - (+220) =$

8) $(-60) - (+6) =$

11) $(+72) - (0) =$

9) $(-13) - (-20) =$

12) $(+432) - (+45) =$

Determine the absolute value:

13) $|-6| =$

14) $|+18| =$

15) $|-186| =$

Find the products:

16) $(60)(-4) =$

19) $26 \cdot 0 =$

17) $(-105)(2) =$

20) $(-20)(10)(-5) =$

18) $(-18)(-4) =$

21) $(18)(-2)(-1)(2) =$

Find the quotients:

22) $\overline{-16}192 =$

25) $\frac{-1056}{-88} =$

23) $0 \div 25 =$

26) $\frac{-256}{32} =$

24) $\frac{108}{-12} =$

27) $95\overline{)9975} =$

Give the sign of each answer without doing the computation:

$$28) \frac{(-35)(-127)(-50)}{(-10)(4)} =$$

$$29) \frac{(-8)(23)(-9)}{5(-6)} =$$

Determine the value of each of the following:

$$30) \frac{(-2)(8)}{24}$$

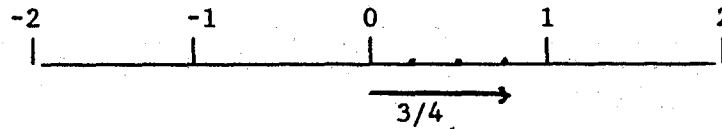
$$31) \frac{(-6)(-4)}{-18}$$

$$32) \frac{(6)(-10)}{16}$$

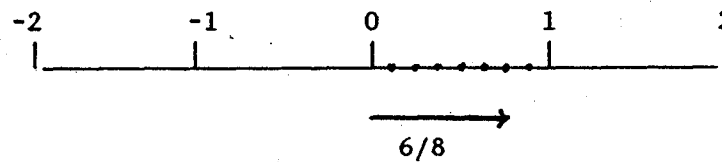
1.1.2 FRACTIONS AND MIXED NUMBERS: RATIONAL NUMBERS

Whole numbers can be divided into equal parts in order to make a measurement more precise. A fraction represents the number of equal parts included in the measurement. For instance, the fraction $3/4$ indicates that there are 4 equal parts to one whole unit and that 3 of the parts are included in the measurement. A fraction may be shown on the number line.

EXAMPLE 1



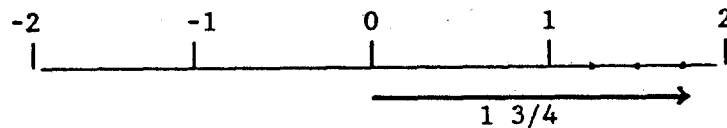
EXAMPLE 2



Example 2 shows the fraction $6/8$ on the number line. It indicates that there are 8 equal parts to one whole unit and that 6 of the parts are included in the measurement. Note that the fractions $3/4$ and $6/8$ represent the same total measurement of the number line. They are therefore EQUIVALENT fractions; however, $3/4$ shows the measurement in its simplest (lowest) terms.

A measurement denoted by the number $1 \frac{3}{4}$ indicates that we are considering one whole unit and 3 or 4 equal parts of another unit.

EXAMPLE 3



A number such as $1 \frac{3}{4}$ is called a MIXED NUMBER, i.e., a whole number and a fraction.

Any number which can be expressed as a fraction is called a RATIONAL NUMBER.

1.1.2.1 MULTIPLICATION OF FRACTIONS

TO FIND THE PRODUCT OF TWO FRACTIONS, MULTIPLY THE TWO NUMERATORS TO FIND THE NUMERATOR OF THE PRODUCT AND MULTIPLY THE TWO DENOMINATORS TO FIND THE DENOMINATOR OF THE PRODUCT.

EXAMPLE 4

$$\frac{5}{7} \cdot \frac{1}{2} = \frac{5 \cdot 1}{7 \cdot 2} = \frac{5}{14} \quad \begin{array}{l} \text{(numerator)} \\ \text{(denominator)} \end{array}$$

EXAMPLE 5

$$\frac{3}{5} \cdot \frac{1}{4} = \frac{3 \cdot 1}{5 \cdot 4} = \frac{3}{20}$$

EXAMPLE 6

$$\left(\frac{2}{3}\right)\left(\frac{3}{8}\right) = \frac{(2)(3)}{(3)(8)} = \frac{6}{24}$$

A negative fraction can be expressed several ways. The minus sign may be placed in front of the numerator, the denominator, or in front of the fraction itself. For example,

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$$

EXAMPLE 7

$$\left(-\frac{2}{3}\right)\left(\frac{5}{7}\right) = \frac{(-2)(5)}{(3)(7)} = \frac{-10}{21} = -\frac{10}{21}$$

EXAMPLE 8

$$\left(-\frac{3}{7}\right)\left(-\frac{2}{5}\right) = \frac{(-3)(-2)}{(7)(5)} = \frac{6}{35}$$

EXERCISES

Find the product of the following:

1) $\frac{1}{2} \times \frac{1}{3} =$

4) $\frac{3}{8} \times -\frac{5}{9} =$

2) $\frac{13}{16} \times \frac{3}{4} =$

5) $\frac{5}{6} \times -\frac{2}{9} =$

3) $-\frac{1}{4} \times \frac{7}{2} =$

6) $\frac{2}{7} \times \frac{3}{4} \times -\frac{1}{2} =$

7) $\frac{7}{64} \times 0 =$

1.1.2.2 DIVISION OF FRACTIONS

TO FIND THE QUOTIENT OF TWO FRACTIONS, INVERT THE DIVISOR AND MULTIPLY AS SHOWN IN SECTION 1.2.1.

NOTE: To invert a fraction is to take the inverse.

The inverse of a number is its RECIPROCAL. THE PRODUCT OF A NUMBER AND ITS RECIPROCAL IS 1.

EXAMPLE 9

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$;

The product of $\frac{2}{3}$ and its reciprocal $\frac{3}{2}$ is

$$\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$$

The reciprocal of 9 is $\frac{1}{9}$; the product of 9 and its reciprocal is

$$\frac{9}{1} \cdot \frac{1}{9} = \frac{9}{9} = 1$$

Another way to state the division rule is:

TO FIND THE QUOTIENT OF TWO FRACTIONS, MULTIPLY THE DIVIDEND BY THE RECIPROCAL OF THE DIVISOR.

EXAMPLE 10

$$\frac{2}{3} \div \frac{3}{7} = \frac{2}{3} \times \frac{7}{3} = \frac{14}{9}$$

EXAMPLE 11

$$\frac{2}{3} \div -\frac{3}{7} = \frac{2}{3} \times -\frac{7}{3} = -\frac{14}{9}$$

EXAMPLE 12

$$-\frac{2}{3} \div -\frac{3}{7} = \frac{2}{3} \times -\frac{7}{3} = \frac{14}{9}$$

EXERCISES

Find the quotients of the following:

1) $\frac{2}{3} \div \frac{1}{2} =$

4) $-\frac{2}{31} \div -\frac{3}{2} =$

2) $\frac{4}{7} \div \frac{3}{4} =$

5) $\frac{9}{32} \div -\frac{2}{9} =$

3) $0 \div \frac{1}{2} =$

6) $\frac{7}{8} \div \frac{3}{8} =$

1.1.2.3 REDUCING FRACTIONS TO SIMPLEST TERMS

Fractions should normally be expressed in their lowest terms or simplest form.

THE VALUE OF A FRACTION IS CHANGED IF ONE TERM IS CHANGED WHILE THE OTHER REMAINS CONSTANT - OR IF A NUMBER OTHER THAN ZERO IS ADDED TO OR SUBTRACTED FROM BOTH TERMS.

THE VALUE OF A FRACTION IS UNCHANGED IF BOTH TERMS ARE MULTIPLIED OR DIVIDED BY THE SAME NUMBER (NOT ZERO).

To express a fraction in its lowest terms, a number must be found that is the largest whole number that can be divided evenly into both numerator and denominator.

EXAMPLE 13

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

$$\frac{9}{9} = \frac{9 \div 9}{9 \div 9} = \frac{1}{1} = 1$$

EXERCISES

Reduce the following fractions to lowest terms:

1) $\frac{3}{9}$

4) $\frac{18}{54}$

2) $\frac{12}{18}$

5) $-\frac{14}{42}$

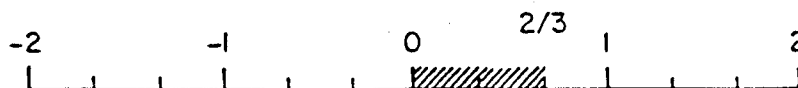
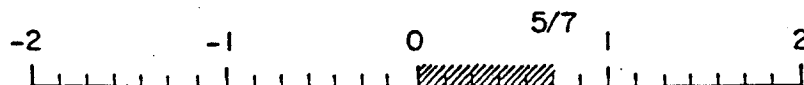
3) $\frac{35}{63}$

6) $\frac{125}{625}$

1.1.2.4 FINDING A COMMON DENOMINATOR

It is not easy to readily determine which of the fractions $\frac{2}{3}$ and $\frac{5}{7}$ is larger. One method is with the number line.

EXAMPLE 14



Likewise, because each unit is divided into a different number of segments (denominators 3 and 7), adding and subtracting the fractions is difficult. The practical method for comparing, adding or subtracting fractions is to change each fraction to an EQUIVALENT value such that its numerator will represent a quantity of equal segments which are common to all the fractions under consideration (COMMON DENOMINATOR). With a common denominator, only the numerators need to be compared, added or subtracted.

TO EXPRESS FRACTIONS IN TERMS OF THEIR COMMON DENOMINATORS, FIRST DETERMINE A QUANTITY THAT IS EXACTLY DIVISIBLE BY EACH OF THE UNLIKE DENOMINATORS. THEN CHANGE EACH FRACTION TO AN EQUIVALENT FRACTION HAVING THIS COMMON DENOMINATOR.

EXAMPLE 15

Find a common denominator of $\frac{2}{3}$ and $\frac{5}{7}$.

$3 \cdot 7 = 21$ Therefore, 21 is exactly divisible by both 3 and 7 and is a

common denominator. Changing each to an equivalent fraction of the common denominator,

$$\frac{2}{3} \cdot \frac{7}{7} = \frac{14}{21} \quad \text{and} \quad \frac{5}{7} \cdot \frac{3}{3} = \frac{15}{21}$$

If the smallest quantity exactly divisible by each of the unlike denominators is used as the common denominator, it is called the **LOWEST COMMON DENOMINATOR (LCD)**.

The lowest common denominator is used primarily when working with more than two fractions. This can be accomplished by a method called factoring.

EXAMPLE: $\frac{5}{6} + \frac{4}{9} + \frac{17}{30}$

by factoring we find they become: $\frac{5}{2 \cdot 3} + \frac{4}{3 \cdot 3} + \frac{17}{2 \cdot 3 \cdot 5}$

Therefore the lowest common denominator = $2 \cdot 3 \cdot 3 \cdot 5 = 90$

Which is easier to work with than $6 \cdot 9 \cdot 30 = 1620$

EXERCISES

Express the following fractions in terms of their lowest common denominator:

1) $\frac{3}{8}, \frac{3}{4}$

ANSWER: $\frac{3}{8}, \frac{6}{8}$

4) $\frac{3}{4}, \frac{2}{3}$

2) $\frac{2}{9}, \frac{1}{2}$

5) $\frac{13}{15}, \frac{8}{9}$

3) $\frac{9}{56}, \frac{7}{8}$

6) $\frac{5}{1}, \frac{2}{9}$

1.1.2.5 ADDING AND SUBTRACTING FRACTIONS

In order to add and subtract fractions, they must have a common denominator. FRACTIONS HAVING UNLIKE DENOMINATORS MAY BE CHANGED TO EQUIVALENT FRACTIONS WITH A COMMON DENOMINATOR. FRACTIONS HAVING LIKE DENOMINATORS MAY BE COMBINED BY ADING OR SUBTRACTING THE NUMERATORS AND PLACING THE SUM/DIFFERENCE OVER THE COMMON DENOMINATOR. REDUCE TO LOWEST TERMS.

EXAMPLE 16

Find the sum of $\frac{2}{3}$ and $\frac{1}{4}$;

in terms of the

common denominator:

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12}$$

(LCD = 12)

$$\frac{1}{4} = \frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$$

combining fractions:

$$\frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$$

EXAMPLE 19

Find the difference of $\frac{8}{12}$ and $\frac{3}{12}$;

$$\frac{8}{12} - \frac{3}{12} = \frac{8-3}{12} = \frac{5}{12}$$

REMEMBER THE LAWS OF SIGNED NUMBERS

EXAMPLE 18

Find the difference of $-1/2$ and $1/3$;

in terms of the
common denominator:
(LCD = 6)

$$-\frac{1}{2} = -\frac{1}{2} \cdot \frac{3}{3} = -\frac{3}{6}$$

$$\frac{1}{3} = \frac{1}{3} \cdot \frac{2}{2} = \frac{2}{6}$$

subtracting fractions:
(change sign of subtra-
hend and add).

$$\left(-\frac{3}{6}\right) - \left(\frac{2}{6}\right) = \left(-\frac{3}{6}\right) + \left(-\frac{2}{6}\right) = \frac{(-3) + (-2)}{6} = -\frac{5}{6}$$

EXERCISES

Add the following fractions and simplify:

1) $\frac{1}{2} + \frac{1}{4}$

4) $\frac{7}{8} + \left(-\frac{1}{3}\right)$

2) $\frac{4}{9} + \frac{13}{18}$

5) $-\frac{3}{5} + \frac{1}{2}$

3) $\frac{6}{7} + \frac{2}{3}$

6) $-\frac{5}{21} + \left(-\frac{11}{42}\right)$

Subtract the following fractions and simplify:

7) $\frac{5}{6} - \frac{1}{24}$

10) $-\frac{1}{2} - \frac{1}{3}$

8) $\frac{7}{11} - \frac{1}{3}$

11) $-\frac{5}{8} - \left(-\frac{1}{4}\right)$

9) $\frac{4}{13} - \frac{1}{6}$

12) $\frac{3}{16} - \left(-\frac{1}{32}\right)$

1.1.2.6 IMPROPER FRACTIONS AND MIXED NUMBERS

An IMPROPER FRACTION is a fraction in which the numerator is equal to or greater than its denominator, such as $7/4$. This form of fraction may result from addition, subtraction, multiplication or division operations. Conversely, this form of fraction may be desired in order to simplify the performance of such operations with mixed numbers.

An improper fraction may be changed to its equivalent mixed number as shown in the following examples:

EXAMPLE 19

$$\frac{7}{4} = \frac{4+3}{4} = \frac{4}{4} + \frac{3}{4} = 1 + \frac{3}{4} = 1\frac{3}{4}$$

EXAMPLE 20

$$\frac{26}{6} = \frac{24+2}{6} = \frac{24}{6} + \frac{2}{6} = 4 + \frac{2}{6} = 4\frac{2}{6} = 4\frac{1}{3}$$

Another method is to simply divide the numerator by the denominator to obtain the whole number, and express the remainder over the denominator as the fraction part of the mixed number, i.e., considering example 19, the denominator (4) divides evenly into the numerator (7) once with a remainder of 3; thus, $1\frac{3}{4}$. Considering example 20; the denominator (6) divides evenly into the numerator (26) four times with a remainder of 2; thus, $4\frac{2}{6}$ which reduces to lowest terms as $4\frac{1}{3}$.

A mixed number may be changed to its equivalent improper fraction as shown in the following examples:

EXAMPLE 21

$$1\frac{7}{8} = \frac{1}{1} + \frac{7}{8} = \left(\frac{1}{1} \cdot \frac{8}{8}\right) + \frac{7}{8} = \frac{8}{8} + \frac{7}{8} = \frac{8+7}{8} = \frac{15}{8}$$

EXAMPLE 22

$$2\frac{3}{4} = \frac{2}{1} + \frac{3}{4} = \left(\frac{2}{1} \cdot \frac{4}{4}\right) + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$$

Another method is to simply multiply the whole number by the denominator and add the numerator to produce the numerator of the improper fraction. Express this value over the original denominator.

EXERCISES

Change the following improper fractions to an equivalent mixed number:

1) $\frac{35}{8}$

3) $\frac{65}{16}$

2) $\frac{29}{9}$

4) $-\frac{18}{7}$

Change the following mixed numbers to their equivalent improper fraction:

5) $6\frac{1}{8}$

7) $21\frac{1}{4}$

6) $9\frac{1}{2}$

7) $-6\frac{4}{7}$

Add the following mixed numbers:

$$9) \quad 1\frac{3}{4} + 2\frac{1}{2} = \frac{7}{4} + \left(\frac{5}{2} \cdot \frac{2}{2}\right) = \frac{7}{4} + \frac{10}{4} = \frac{7+10}{4} = \frac{17}{4} = 4\frac{1}{4}$$

$$10) \quad 2\frac{7}{8} + 5\frac{1}{9} =$$

$$11) \quad 8\frac{1}{3} + 1\frac{1}{2} =$$

$$12) \quad 1\frac{15}{16} + 9\frac{1}{4} =$$

Subtract the following mixed numbers:

$$13) \quad 6\frac{1}{2} - 4\frac{1}{3} =$$

$$14) \quad 16\frac{1}{4} - 3\frac{1}{8} =$$

$$15) \quad 2\frac{1}{7} - 1\frac{3}{4} =$$

$$16) \quad 1\frac{3}{4} - \left(-\frac{7}{8}\right) =$$

Find the product of the following mixed numbers:

$$17) \quad 8\frac{1}{2} \times 2\frac{1}{3} =$$

$$18) \quad -1\frac{1}{4} \times 1\frac{1}{16} =$$

Find the quotient of the following mixed numbers:

$$19) \quad 3\frac{1}{3} \div 1\frac{1}{3} =$$

$$20) \quad -1\frac{1}{8} \div \frac{1}{4} =$$

$$21) \quad 6\frac{3}{4} \div 0 =$$

1.1.2.7 COMPLEX FRACTIONS

A fraction whose numerator, denominator or both contain other fractions is a **COMPLEX FRACTION**. Complex fractions are shown in the following example.

EXAMPLE 23

$$\frac{\frac{1}{4}}{2}, \quad \frac{3}{\frac{1}{2}}, \quad \frac{\frac{1}{3}}{\frac{1}{2}}$$

The line separating numerator and denominator of the expression may be considered as a symbol for division. The expressions may then be solved using the normal rules for dividing fractions.

EXAMPLE 24

$$\frac{\frac{1}{4}}{2} = \frac{1}{4} \div 2 = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$\frac{3}{\frac{1}{2}} = 3 \div \frac{1}{2} = \frac{3}{1} \cdot \frac{2}{1} = \frac{6}{1} = 6$$

$$\frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$$

Another method of solving complex fractions, which will become more important in the study of algebra, is to choose a fractional symbol for 1 whose numerator and denominator are the least common multiple of all the denominators of the expression. Multiply that fraction by the complex fraction, then simplify.

EXAMPLE 25

$$\frac{\frac{1}{4}}{2} = \left(\frac{4}{4}\right) \cdot \left(\frac{\frac{1}{4}}{2}\right) = \frac{4 \cdot 1}{4 \cdot 2} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{7 + \frac{5}{9}}{6 - \frac{5}{6}} = \left(\frac{18}{18}\right) \left(\frac{7 + \frac{5}{9}}{6 - \frac{5}{6}}\right) = \frac{18\left(7 + \frac{5}{9}\right)}{18\left(6 - \frac{5}{6}\right)} = \frac{18(7) + 18\left(\frac{5}{9}\right)}{18(6) - 18\left(\frac{5}{6}\right)} = \frac{126 + 10}{108 - 15} = \frac{136}{93}$$

$$\frac{\frac{2}{5} - \frac{3}{4}}{2 + \frac{1}{2}} = \left(\frac{20}{20}\right) \left(\frac{\frac{2}{5} - \frac{3}{4}}{2 + \frac{1}{2}}\right) = \frac{20\left(\frac{2}{5} - \frac{3}{4}\right)}{20\left(2 + \frac{1}{2}\right)} = \frac{20\left(\frac{2}{5}\right) - 20\left(\frac{3}{4}\right)}{20(2) + 20\left(\frac{1}{2}\right)} = \frac{8 - 15}{40 + 10} = \frac{-7}{50}$$

EXERCISES

Simplify the following complex fractions:

$$1) \frac{\frac{4}{1}}{\frac{1}{8}}$$

$$2) \frac{\frac{1}{9}}{\frac{6}{6}}$$

$$3) \frac{\frac{5}{8}}{\frac{1}{3}}$$

$$4) \frac{\frac{3}{5}}{\frac{1}{4}}$$

$$5) \frac{2\frac{1}{2}}{\frac{2}{3}}$$

$$6) \frac{\frac{5}{12}}{\frac{1}{8}}$$

1.1.2.8 PRACTICE PROBLEMS

Find the following products and express in simplest terms.

1) $\frac{3}{4} \times \frac{4}{5} =$

3) $\frac{4}{9} \times \frac{7}{8} =$

2) $\frac{1}{2} \times \frac{3}{7} =$

4) $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} =$

Find the following quotients and express in simplest terms.

5) $\frac{5}{16} \div \frac{1}{4} =$

7) $\frac{5}{8} \div \frac{1}{3} =$

6) $\frac{4}{11} \div 3 =$

8) $\frac{9}{13} \div \frac{1}{2} =$

Find the following sums and express in simplest terms.

9) $\frac{2}{5} + \frac{4}{9} =$

11) $\frac{2}{7} + \frac{2}{3} =$

10) $\frac{5}{6} + \frac{3}{4} =$

12) $\frac{3}{8} + \frac{5}{16} + \frac{3}{4} =$

Find the following differences and express in simplest terms.

13) $\frac{5}{9} - \frac{1}{2} =$

15) $\frac{9}{16} - \frac{5}{32} =$

14) $\frac{3}{10} - \frac{1}{30} =$

16) $\frac{6}{7} - \frac{3}{4} =$

Perform the indicated operations and express all answers in simplest terms.

17) $2\frac{1}{3} \cdot \frac{1}{2} =$

23) $\frac{\frac{8}{21}}{\frac{2}{7}} =$

18) $\frac{5}{7} - \left(-\frac{1}{5}\right) =$

24) $\frac{\frac{1}{16}}{\frac{1}{64}} =$

19) $15\frac{1}{3} + 2\frac{1}{2} =$

25) $\frac{\frac{x}{y}}{\frac{x}{y}} =$

20) $3\frac{1}{3} - 1\frac{1}{4} =$

26) $\left(\frac{1\frac{5}{8}}{\frac{3}{4}}\right) \left(\frac{\frac{2}{3}}{\frac{1}{2}}\right) =$

21) $-\frac{5}{6}x - \frac{2}{7} =$

27) $\frac{4\frac{1}{2}}{\frac{7}{9}} + \frac{\frac{3}{7}}{\frac{1}{9}} =$

22) $23\frac{1}{8} \times 0 =$

28) $\frac{5 - \frac{3}{4}}{4 + \frac{2}{3}} =$

29) $\frac{\frac{3}{5} - \frac{5}{6}}{2 - \frac{2}{3}} =$

1.1.3 DECIMALS AND PERCENTAGE

The Decimal System is based on the number 10. The ordinary numerals used for whole numbers are DECIMAL NUMERALS. To show the meaning of decimal numerals, or DECIMAL NOTATION, we can write expanded numerals as shown in the following examples.

EXAMPLE 1

$$1975 = 1(1000) + 9(100) + 7(10) + 5(1) = 1000 + 900 + 70 + 5$$

The value of each position is ten times that to its right.

Fractions and mixed numbers may also be written as decimals. Since a fraction is less than one whole unit of measurement, each position is one tenth that to its left.

EXAMPLE 2

$$\begin{aligned} .125 &= 1 \cdot \left(\frac{1}{10}\right) + 2 \cdot \left(\frac{1}{100}\right) + 5 \cdot \left(\frac{1}{1000}\right) = \frac{1}{10} + \frac{2}{100} + \frac{5}{1000} = \\ &= \frac{100}{1000} + \frac{20}{1000} + \frac{5}{1000} = \frac{125}{1000} \text{ (common denominator)} \end{aligned}$$

EXAMPLE 3

$$1975.125 = 1000 + 900 + 70 + 5 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$$

A period, called a DECIMAL POINT, is placed to the right of the last digit in a whole number and to the left of the first digit in a fraction. A number shown without a decimal point is assumed to be a whole number (decimal point to the right of the last digit).

1.1.3.1 ADDING AND SUBTRACTING DECIMALS

Adding or subtracting with decimals is similar to adding or subtracting with numerals for whole numbers. The important difference is the decimal point.

WHEN ADDING OR SUBTRACTING DECIMALS, THE NUMERALS MUST BE WRITTEN WITH THEIR DECIMAL POINTS ALIGNED IN THE SAME VERTICAL COLUMN.

EXAMPLE 4

Find the sum of 1.125 and .25

$$\begin{array}{r} 1.125 \\ + 0.250 \\ \hline 1.375 \end{array}$$

EXAMPLE 5

Find the difference of 1.125 and .25

$$\begin{array}{r} 1.125 \\ -0.250 \\ \hline 0.875 \end{array}$$

It is helpful to insert extra 0's so that each of the numerals are written with the same number of decimal places.

EXERCISES:

Perform the indicated operations

- | | |
|--------------------|-----------------------|
| 1) 13.186 + 25.01 | 4) 1526 - 1516.18 |
| 2) 49.106 + 75.954 | 5) 17 + (-8.5) |
| 3) 85.1 - 16.85 | 6) -48.44 - (+ 18.21) |

1.1.3.2 MULTIPLYING DECIMALS

Finding the product of two decimal numerals is similar to finding the product of two numerals for whole numbers. The important difference is the decimal point.

THE NUMBER OF FIGURES TO THE RIGHT OF THE DECIMAL POINT IN A PRODUCT IS THE SUM OF THE NUMBER OF FIGURES TO THE RIGHT OF THE DECIMAL POINT IN BOTH MULTIPLICAND AND MULTIPLIER.

EXAMPLE 6

$$\begin{array}{r} 37.4 \\ \times 4.51 \\ \hline 374 \\ 1870 \\ 1496 \\ \hline 168.674 \end{array}$$

FIGURES TO THE RIGHT OF DECIMAL
(MULTIPLICAND : 1)
(MULTIPLIER : 2)
(PRODUCT : 3)

1.1.3.3 DIVIDING DECIMALS

Finding the quotient of two decimal numerals is similar to finding the quotient of two numerals for whole numbers. The important difference is the decimal point. To locate the decimal point in a quotient, the following example is used:

EXAMPLE 6

Find the quotient of $6.5 \div 1.25$

- a. Make the divisor a whole number by moving its decimal point to the right. Then move the decimal point of the dividend the same number of places to the right.

$$\begin{array}{r} 1.25 \) \ 6.50 \\ \hline \end{array}$$

NOTE: We actually multiplied both divisor (denominator) and dividend (numerator) by 100. Therefore the value of the fraction

$$\frac{6.5}{1.25} \text{ has not changed. } \left(\frac{6.5}{1.25} \times \frac{100}{100} = \frac{650}{125} \right).$$

- b. Divide in the same manner, as for whole numbers and locate the decimal point in the quotient directly above the decimal point of the dividend.

$$\begin{array}{r} 5.2 \\ 125 \overline{) 650.0} \\ \underline{625} \\ 250 \\ \underline{250} \\ 0 \end{array}$$

As many 0's as necessary may be annexed to the dividend, TO THE RIGHT OF THE DECIMAL POINT, to carry out the operation.

EXERCISES:

Perform the indicated operations

- | | |
|-----------------------|-----------------------|
| 1) 16.1×8.5 | 4) $132.5 \div 25$ |
| 2) 12.23×8.9 | 5) $43.365 \div 8.26$ |
| 3) 32.187×3 | 6) $1250 \div 2.5$ |

1.1.3.4 CHANGING FRACTIONS TO DECIMAL NOTATION

Any fraction may be expressed in decimal notation by dividing THE NUMERATOR BY THE DENOMINATOR.

EXAMPLE 7

$$\frac{1}{10} = 1 \div 10 = 10 \overline{) \begin{array}{r} .1 \\ 10 \end{array}} \text{ and } \frac{1}{100} = 1 \div 100 = 100 \overline{) \begin{array}{r} .01 \\ 000 \\ 100 \\ 100 \end{array}}$$

Note that when the denominator is an exact multiple (power) of ten, the decimal point appears one place to the left of the numeral for each multiple.

EXAMPLE 8

$$\frac{3}{8} = 3 \div 8 = \begin{array}{r} .375 \\ 8 \overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Some fractions will always have a remainder when the division is performed and therefore the decimal will only be an approximation.

EXAMPLE 9

$$\frac{2}{3} = 2 \div 3 = \begin{array}{r} .6666 \\ 3 \overline{)2.0000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

To express the fraction 2/3 in decimal notation carried out to three decimal places, we ROUND-OFF the final digit, i.e., 2/3 = .667.

1.1.3.5 SIGNIFICANT FIGURES

The precision of a measurement is expressed by the number of significant figures in the reading. If you say the length of a control rod is 40.1 inches, you imply that you measured it to the nearest tenth of an inch. Similarly, if you state the measurement as 40.10 inches, you imply that you measured it to the nearest hundredth of an inch.

A SIGNIFICANT DIGIT IS A DIGIT OBTAINED BY MEASUREMENT.

Some rules for determining significant digits are:

- (1) All non-zero digits are significant.

EXAMPLE 10

125 has three significant figures 1, 2 and 5.

- (2) Zeroes located between non-zero figures are significant.

EXAMPLE 11

401 has three significant figures 4, 0 and 1.

- (3) Zeroes are NOT significant if they are used merely to show the location of the decimal point--UNLESS OTHERWISE STATED.

EXAMPLE 12

4100 has two significant figures 4 and 1.

EXAMPLE 13

0.041 has two significant figures 4 and 1.

- (4) Zeroes to the right of BOTH a decimal and a non-zero figure are significant.

EXAMPLE 14

.410 has three significant figures 4, 1 and zero.

Using the above rules, we interpret the following measurements:

400 miles = measurement in hundreds of miles with 1 digit accuracy to the nearest hundred (miles), UNLESS OTHERWISE STATED.

410 miles = measurement in hundreds of miles with 2 digit accuracy to the nearest ten (miles), UNLESS OTHERWISE STATED.

401 miles = measurement in hundreds of miles with 3 digit accuracy to the nearest one (mile).

401.1 miles = measurement in hundreds of miles with 4 digit accuracy to the nearest tenth (mile).

401.01 miles = measurement in hundreds of miles with 5 digit accuracy to the nearest hundredth (mile).

401.100 miles = measurement in hundreds of miles with 6 digit accuracy to the nearest thousandth (mile).

In calculating measurements, we often desire to decrease the total number of significant figures in order to simplify the calculation. This process is called ROUNDING-OFF a number.

IF THE FIGURE TO THE RIGHT OF THE DESIRED LEAST SIGNIFICANT DIGIT IS 0, 1, 2, 3, or 4, CONSIDER THE FIGURE ZERO AND DROP THE REMAINDER OR CHANGE IT TO ZERO. IF THE FIGURE TO THE RIGHT OF THE DESIRED LEAST SIGNIFICANT DIGIT IS 5, 6, 7, 8 or 9, THEN INCREASE THE DESIRED LEAST SIGNIFICANT DIGIT BY ONE AND DROP THE REMAINDER OR CHANGE IT TO ZERO.

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EO3: ADD, SUBTRACT, MULTIPLY, AND DIVIDE REAL NUMBERS AND EXPRESS THE ANSWERS IN THE PROPER NUMBER OF SIGNIFICANT FIGURES.

1.1.3.5 Significant Figures

In science, many of the numbers that will be dealt with are measurements. In mathematics, the number $12 = 12.000000$ etc., but if 12 represents a measurement, then 12 does not equal 12.0 because the number 12.0 represents a precision that the number 12 does not. The precision of a measurement is indicated by the number of significant figures the number has. An example will illustrate. Suppose the width of a room was measured and the value was 6.783 meters. The measurement assures us that the width of the room is greater than 6 meters. It, also, assures us that the width is greater than 78 centimeters, but less than 79 centimeters on the 7th meter. The number 3 in the measurement indicates that the person who measured the room estimated the width of the room to be $\frac{3}{10}$ of the way along the 79 cm of the 7th meter. This measurement has 4 significant figures. In general, all the known digits and the digit that represents the first estimate are the significant figures of the measurement.

A SIGNIFICANT DIGIT IS A DIGIT OBTAINED BY MEASUREMENT

Some rules for determining significant digits are:

- (1) All non-zero digits are significant.
- (2) Zeroes located between non-zero figures are significant.
- (3) Zeroes are not significant if they are used merely to show the location of the decimal point (unless otherwise stated.)
- (4) Zeroes to the right of both a decimal and a non-zero figure are significant.

EXAMPLE 10: 125 has three significant figures 1, 2 and 5.

EXAMPLE 11: 401 has three significant figures 4, 0 and 1.

EXAMPLE 12: 4100 has two significant figures 4 and 1.

EXAMPLE 13: 0.041 has two significant figures 4 and 1.

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EXAMPLE 14: 0.410 has three significant figures 4, 1 and zero. Using the above rules, we interpret the following measurements:

400 miles.....measurement in hundreds of miles with 1 digit accuracy to the nearest hundred (miles), *unless otherwise stated.*

410 miles.....measurement in hundreds of miles with 2 digit accuracy to the nearest ten (miles), *unless otherwise stated.*

401 miles.....measurement in hundreds of miles with 3 digit accuracy to the nearest one (mile).

401.1 miles.....measurement in hundreds of miles with 4 digit accuracy to the nearest tenth (mile).

401.01 miles.....measurement in hundreds of miles with 5 digit accuracy to the nearest hundredth (mile).

401.100 miles.....measurement in hundreds of miles with 6 digit accuracy to the nearest thousandth (mile).

In calculating measurements, we often desire to decrease the total number of significant figures in order to simplify the calculation. This process is called *rounding-off* a number.

If the figure to the right of the desired least significant digit is 0, 1, 2, 3, or 4, consider the figure zero and drop the remainder or change it to zero. If the figure to the right of the desired least significant

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EXAMPLE 15: Round each measurement to two significant digits:

	NUMBER	ROUNDED
a)	2634	2600
b)	266.3	270
c)	26.25	26
d)	0.2697	0.27

Notice that in (a) it was necessary to place zeroes in the hundreds and units positions, and in part (b) a zero was necessary in the units position. These zeroes, though they are not significant, are necessary to indicate the location of the understood decimal point. NO zeroes may be placed in the solution to (c), because 26.0 would indicate three significant digits, not two. Similarly, in (d) placement of any zeroes would indicate more than two significant digits.

The sum or difference of measurements is only as precise as the measurement having the least precise place value. For example, if the measurement having the least precise place value in the sum of several measurements is 6.7 millimeters, then the sum should be rounded off to the nearest tenth of a millimeter.

The product or quotient of measurements is only as precise as the least precise measurement. If the multiplication or division is one of a series, one more digit may be carried in the intermediate products or quotients than will be recorded in the answer.

EXAMPLE 17	
	3.27
	41.36
+	125.2
	169.83
Answer:	169.8

EXAMPLE 15

Round each measurement to two significant digits:

	<u>NUMBER</u>	<u>ROUNDED</u>
a)	2634	2600
b)	266.3	270
c)	26.25	26
d)	.2697	.27

Notice that in (a) it was necessary to place zeroes in the hundreds and units positions, and in part (b) a zero was necessary in the units position. These zeroes, though they are not significant, are necessary to indicate the location of the understood decimal point. NO zeroes may be placed in the solution to (c), because 26.0 would indicate three significant digits, not two. Similarly, in (d) placement of any zeroes would indicate more than two significant digits.

1.1.3.6 PERCENTAGE

The word percent means PER HUNDRED and is expressed using the percent symbol: %.

TO CONVERT A DECIMAL NUMERAL TO PERCENT, MULTIPLY THE NUMERAL BY 100 AND ADD THE PERCENT SYMBOL.

EXAMPLE 16

$$\begin{aligned} .03 \times 100 &= 3\% \\ .3 \times 100 &= 30\% \\ .313 \times 100 &= 31.3\% \end{aligned}$$

TO CONVERT PERCENT TO DECIMAL NOTATION, DIVIDE BY 100 AND REMOVE THE PERCENT SYMBOL.

EXAMPLE 17

$$\begin{aligned} 5\% \div 100 &= .05 \\ 50\% \div 100 &= .5 \\ 51.5\% \div 100 &= .515 \end{aligned}$$

TO CONVERT A FRACTION TO PERCENT, DIVIDE THE NUMERATOR BY THE DENOMINATOR. THEN MULTIPLY THE QUOTIENT BY 100 AND ADD A PERCENT SYMBOL.

EXAMPLE 18

3 is what percent of 8?

$$\frac{3}{8} = 3 \div 8 = .375$$
$$.375 \times 100 = 37.5\%$$

TO FIND THE PRODUCT OF PERCENTAGE AND ANOTHER NUMBER, FIRST CHANGE PERCENTAGE TO ITS DECIMAL EQUIVALENT AND THEN MULTIPLY.

EXAMPLE 19

Find 40% of 785.

$$40\% = \frac{40}{100} = .4, \text{ then } \begin{array}{r} 785 \\ \times .4 \\ \hline 314.0 \end{array}$$

EXAMPLE 20

A pump is designed to deliver 8000 gallons per minute. If the pump is presently delivering 3500 gallons per minute, what percent of design flow is the pump providing?

$$\frac{3500}{8000} = .4375$$
$$.4375 \times 100 = 43.75\%$$

EXAMPLE 21

When the plant is at 50% power, xenon worth is 85% of full power xenon worth. If full power xenon worth is 2780, what is xenon worth when the plant is at 50% power?

$$\begin{aligned} \text{worth} &= 85\% \text{ of } 2780 \\ &= .85 \times 2780 \\ &= 2363 \end{aligned}$$

EXERCISES:

1. Indicate the number of significant figures in each of the following.

- a) 42,575 _____ e) 42.110 _____
b) 1305 _____ f) .00015 _____
c) 1.25 _____ g) 1.09 _____
d) 86,000,000 _____ h) 0.10 _____

2. Express the following fractions in decimal notation.

- a) $\frac{3}{4}$
b) $\frac{1}{8}$
c) $\frac{5}{16}$
d) $\frac{9}{32}$

3. Express the following in decimal notation and round-off to three significant figures.

- a) $\frac{1}{3}$
b) $85 + 26$
c) 14.25×5.32
d) 21×931

4. Convert the fraction $\frac{5}{8}$ to percentage.

5. What is 33% of 1750?

6. One cubic foot of air is composed of approximately .21 cubic feet of oxygen, .78 cubic feet of nitrogen and .01 cubic feet of argon, carbon dioxide and other gasses. Express these values as percent of the total mixture.

1.1.3.7 PRACTICE PROBLEMS

ADDITION OF DECIMALS

1)
$$\begin{array}{r} 83.9 \\ 7.6 \\ \hline 126.3 \end{array}$$

3) $0.014 + 0.971 + 2.360$

2)
$$\begin{array}{r} 1.074 \\ 0.985 \\ \hline 0.977 \end{array}$$

4) $0.00274 + 0.000922 =$

SUBTRACTION OF DECIMALS

5)
$$\begin{array}{r} 931.6 \\ 726.5 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 429.730 \\ 7.568 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 10.00 \\ 8.98 \\ \hline \end{array}$$

8) $0.00648 - 0.000549 =$

MULTIPLICATION OF DECIMALS

9)
$$\begin{array}{r} 54.6 \\ 12 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 3.92 \\ 4.8 \\ \hline \end{array}$$

11)
$$\begin{array}{r} 0.648 \\ 0.095 \\ \hline \end{array}$$

12)
$$\begin{array}{r} 0.913 \\ 0.26 \\ \hline \end{array}$$

DIVISION OF DECIMALS

13) $7.1 \overline{) 68.16}$

15) $6.52 \overline{) 255.584}$

14) $45 \div 0.15 =$

16) $0.1015503 \div 0.219 =$

MULTIPLY AND ROUND-OFF THE PRODUCTS TO THREE SIGNIFICANT FIGURES.

17) 42×546

18) 6.0281×33

19) 1.005×5

- 20) Assume the normal operating temperature of a reactor's cooling system is 500°F. If we desire to reduce the temperature to a value 85% of normal, what will be the new temperature?
- 21) If the maximum resistance measured by a resistance-temperature detector is 750 ohms and the detector is presently measuring 450 ohms, what % of its range is indicated?
- 22) Approximately 99.3% of natural Uranium is of a specific type referred to as U238. If a nuclear reactor contains 135,000 pounds of natural Uranium, how many pounds of U238 does it contain?
- 23) A certain reactor is capable of producing 3423 Megawatts of thermal energy. If only 1129 Megawatts can be transformed into electrical power, what percentage of the total capability remains unused?

1.1.4 EXPONENTIAL NOTATION

Working with large numbers can be simplified by the use of EXPONENTS. An exponent is written above and to the right of a base number. Numbers which are multiplied together are called FACTORS of the product; the EXPONENT SHOWS HOW MANY TIMES THE BASE IS USED AS A FACTOR. A POWER is a number which can be expressed by a base and exponent.

EXAMPLE 1

$(4)(4)$ may be written as $(4)^2$, where 4 is the base, 2 is the exponent and $(4)^2$ is the second power of 4 or the square of 4.

EXAMPLE 2

The following examples show the meaning of exponential notation using positive integer exponents.

<u>Symbol</u>	<u>Name</u>	<u>Equivalent</u>
2^1	Two to the first power	$(2) = 2$
2^2	Two to the second power or two squared	$(2)(2) = 4$
2^3	Two to the third power or two cubed	$(2)(2)(2) = 8$
2^4	Two to the fourth power	$(2)(2)(2)(2) = 16$
2^5	Two to the fifth power	$(2)(2)(2)(2)(2) = 32$

A NUMBER TO THE FIRST POWER (Exponent 1) IS EQUAL TO THAT NUMBER. i.e., the number is used as a factor one time.

EXERCISES

Express the following using bases and exponents:

1) $(4)(4)(4)(4)(4)(4) =$

2) $(6) =$

3) $(16)(16)(16) =$

4) $(x)(x)(x)(x) =$

5) $(3)(3)(5)(5)(5) =$

1.1.4.1 ROOTS

The **ROOT** of a given number is one of the equal factors of that number. For example, the square root of four is two, since two squared equals four or $2^2 = 4$. When the root of a number is desired, a symbol called a **RADICAL** is used. The number within the radical is called the **RADICAND**.

EXAMPLE 3

$$\sqrt{4} = \text{square root of } 4 = 2$$

$$\sqrt[3]{8} = \text{cube root of } 8 = 2$$

$$\sqrt[4]{16} = \text{fourth root of } 16 = 2$$

$$\sqrt[5]{32} = \text{fifth root of } 32 = 2$$

Note that the root desired is denoted by a superscript or **INDEX** to the left of the radical. If a radical has no index, it is assumed to indicate the square root.

EXERCISES

Find the following roots using your calculator.

(1) $\sqrt{64}$

(2) $\sqrt[3]{95}$

(3) $\sqrt[3]{18}$

(3) $\sqrt{68}$

1.1.4.2 LAWS OF EXPONENTS

EXAMPLE 4

Consider the product $2^3 \cdot 2^2$. Since $2^3 = 2 \cdot 2 \cdot 2$ and $2^2 = 2 \cdot 2$,

$$2^3 \cdot 2^2 = \overbrace{(2 \cdot 2 \cdot 2)}^{3 \text{ factors}} \overbrace{(2 \cdot 2)}^{2 \text{ factors}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ factors}} = 2^{3+2} = 2^5.$$

More generally,

$$a^m \cdot a^n = \overbrace{(a \cdot a \cdot \dots \cdot a)}^{m \text{ factors}} \overbrace{(a \cdot a \cdot \dots \cdot a)}^{n \text{ factors}} = a^{m+n}$$

(1) FOR ALL REAL NUMBERS m AND n , $a^m \cdot a^n = a^{m+n}$
--

Rule (1) states that to multiply powers having the same base, keep the base and add the exponents.

EXERCISES

Express the answers to the following using exponential notation.

1) $16^5 \cdot 16^7$

2) $7^4 \cdot 7$

3) $5^3 \cdot 5^2 \cdot 5^5$

EXAMPLE 5

Consider the quotient $\frac{2^5}{2^2}$.

$$\frac{2^5}{2^2} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = \frac{2}{2} \cdot \frac{2}{2} \cdot 2 \cdot 2 \cdot 2 = 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 = 2^3.$$

$$\frac{2^5}{2^2} = 2^{5-2} = 2^3.$$

More generally, for $m > n$,

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot a \cdot \dots \cdot a}^{m \text{ factors}}}{\underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}} = \frac{\overbrace{(a \cdot a \cdot \dots \cdot a)}^{n \text{ factors}} \cdot \overbrace{(a \cdot a \cdot \dots \cdot a)}^{m-n \text{ factors}}}{\underbrace{(a \cdot a \cdot \dots \cdot a)}_{n \text{ factors}}} = a^{m-n}$$

(2) FOR ALL REAL NUMBERS m AND n , $\frac{a^m}{a^n} = a^{m-n}$

Rule (2) states that to divide powers with the same base, keep the base and subtract the exponents.

EXERCISES

Express the answers to the following in exponential notation.

1) $\frac{6^7}{3^6}$

2) $\frac{5^{17}}{11^5}$

3) $\frac{2^{10}}{5^2}$

EXAMPLE 6

Consider the quotient $\frac{5^3}{3^5}$.

$\frac{5^3}{3^5} = \frac{125}{125} = 1$, and by the division rule, $\frac{5^3}{3^5} = 5^{3-3} = 5^0$.

Therefore, $5^0 = 1$. In fact the same procedure can be used to show that any number (except zero) with a zero exponent is equal to one, that is

$(3) \quad a^0 = 1, \text{ FOR ALL } a \text{ EXCEPT ZERO}$

EXAMPLE 7

Consider $(2^3)^2$. By the definition of exponent, $(2^3)^2 = 2^3 \cdot 2^3$. By the multiplication rule for exponents, $2^3 \cdot 2^3 = 2^{3+3} = 2^6$. Therefore, $(2^3)^2 = 2^{3 \cdot 2} = 2^6$.

More generally $(a^m)^n = \underbrace{a^m \cdot a^m \cdot \dots \cdot a^m}_{n \text{ factors}} =$

$$\underbrace{(a \cdot a \cdot \dots \cdot a)_{m \text{ factors}} (a \cdot a \cdot \dots \cdot a)_{m \text{ factors}} \cdot \dots \cdot (a \cdot a \cdot \dots \cdot a)_{m \text{ factors}}}_{n \text{ parentheses}} = a^{m \cdot n}$$

(4) FOR ALL REAL NUMBERS m AND n , $(a^m)^n = a^{m \cdot n}$

Note the difference between rule (1) and rule (4). In rule (1), we are multiplying a power by a power, and we add the exponents. In rule (4), we are taking a power of a power, and we multiply the exponents.

EXERCISES

Express the answers to the following in exponential notation.

1) $(3^7)^4$

2) $(2^5)^5$

3) $(5^{11})^2$

EXAMPLE 8

Consider $(\frac{3}{4})^3$. Since $(\frac{3}{4})^3 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$, and since the rule for multiplying

fractions says that $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3 \cdot 3}{4 \cdot 4 \cdot 4} = \frac{3^3}{4^3}$, it follows that $(\frac{3}{4})^3 = \frac{3^3}{4^3}$.

More generally, $(\frac{a}{b})^n = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \dots \cdot \frac{a}{b}}_{n \text{ factors}} = \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{n \text{ factors}}}{\underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ factors}}} = \frac{a^n}{b^n}$.

(5) FOR ALL REAL NUMBERS n , $(\frac{a}{b})^n = \frac{a^n}{b^n}$

EXERCISES

Express answers to the following in exponential notation (Remove parentheses).

1) $\left(\frac{3}{5}\right)^7$

2) $\left(\frac{2}{7}\right)^6$

3) $\left(\frac{5}{9}\right)^8$

EXAMPLE 9

Consider $\frac{3^4}{3^6}$. Since $\frac{3^4}{3^6} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \left(\frac{3}{3}\right)\left(\frac{3}{3}\right)\left(\frac{3}{3}\right)\left(\frac{1}{3 \cdot 3}\right)$

$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{1}{3 \cdot 3} = \frac{1}{3^2}$, it follows that $\frac{3^4}{3^6} = \frac{1}{3^2}$. If we apply the

division rule for exponents, $\frac{3^4}{3^6} = 3^{4-6} = 3^{-2}$. It follows, therefore, that

$3^{-2} = \frac{1}{3^2}$. More generally,

(6) FOR ALL NON-ZERO BASE a , $a^{-n} = \frac{1}{a^n}$

EXERCISES

Express answers to the following in exponential notation with positive exponents.

1) 7^{-5}

2) 3^{-2}

3) 5^{-1}

EXAMPLE 10

Consider $\sqrt{5}$. By the definition of square root, $\sqrt{5}$ is that number which when multiplied by itself is equal to 5, that is,

$$\sqrt{5} \cdot \sqrt{5} = 5.$$

Also, from the multiplication rule for exponents,

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2} + \frac{1}{2}} = 5^1 = 5.$$

Therefore, $\sqrt{5} \cdot \sqrt{5} = 5 = 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$. It follows that

$$\sqrt{5} = 5^{\frac{1}{2}}$$

Similarly,

$$\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = 5 = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}}, \text{ so}$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}},$$

and

$$\sqrt[4]{5} \cdot \sqrt[4]{5} \cdot \sqrt[4]{5} \cdot \sqrt[4]{5} = 5 = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}}, \text{ so}$$

$$\sqrt[4]{5} = 5^{\frac{1}{4}}.$$

In general,

$$(7) \quad \sqrt[n]{a} = a^{\frac{1}{n}}$$

Rule (7) states that any radical can be expressed in exponential notation by raising the number under the radical to the reciprocal of the index of the radical.

EXERCISES

1) Express each radical in exponential notation.

a) $\sqrt[4]{11}$

b) $\sqrt{2}$

c) $\sqrt[5]{17}$

2) Express each power as a radical.

a) $7^{\frac{1}{3}}$

b) $7^{\frac{1}{2}}$

c) $2^{\frac{1}{4}}$

3) Find the following roots using your calculator.

a) $13^{\frac{1}{3}}$

b) $52^{\frac{1}{2}}$

c) $18^{\frac{1}{3}}$

Summarizing the rules of exponents thus far obtained:

$$\begin{array}{l} 1) a^m \cdot a^n = a^{m+n} \\ 2) \frac{a^m}{a^n} = a^{m-n} \\ 3) a^0 = 1 \\ 4) (a^m)^n = a^{m \cdot n} \\ 5) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \\ 6) a^{-n} = \frac{1}{a^n} \\ 7) \sqrt[n]{a} = a^{\frac{1}{n}} \end{array}$$

EXERCISES

Use the appropriate laws of exponents to express the answer to each of the following with a base to a positive exponent.

1) $16^2 \cdot 16^3$

7) $47^5 + 47^5$

12) $(2^{-2})^3$

2) $13^2 \cdot 13$

8) $7^6 \cdot 7^{-6}$

3) $21^8 + 21^4$

4) $5^3 + 5^5$

9) $\sqrt[5]{12}$

5) $11^{-6} \cdot 11^3$

10) $3^3 + 3^{-3}$

6) $2^2 \cdot \frac{1}{2^3}$

11) $\frac{2^4 \cdot 2^5}{2^7}$

1.1.4.3 SCIENTIFIC NOTATION

Standard Scientific Notation is a method of expressing numbers in powers of ten in order to simplify computations. In SCIENTIFIC NOTATION a number is written as the product of two factors: one factor is a number between 1 and 10; the other factor is a power of 10.

Combining decimal notation and the laws of exponents, we know that;

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$$

$$10^{-1} = \frac{1}{10} = .1$$

$$10^{-2} = \frac{1}{10 \cdot 10} = .01$$

$$10^{-3} = \frac{1}{10 \cdot 10 \cdot 10} = .001$$

$$10^{-4} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10} = .0001$$

$$10^{-5} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = .00001$$

$$10^{-6} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = .000001$$

Each place represents a power to ten. A change of one in the exponent represents a change in the place value of each digit.

Note that with negative exponents, the number itself is not negative, but is between 0 and 1.

Scientific Notation uses two FACTORS: a number between 1 and ten multiplied by a power of ten.

EXAMPLE 11

186,000 miles per second = 1.86×10^5 miles per second

602,000,000,000,000,000,000,000 atoms = 6.02×10^{23} atoms

(The number of places the decimal is moved to the left = positive exponent)

EXAMPLE 12

.000000006 centimeters = 6×10^{-9} centimeters

.0000000000001 centimeters = 1×10^{-13} centimeters

(The number of places the decimal is moved to the right = negative exponent)

EXERCISES

Express the following numbers using scientific notation.

a) 36,000 =

c) .000006 =

b) 5,140,000 =

d) .00125 =

Express the following numbers using decimal notation.

a) 1.5×10^6 =

d) 1×10^{-3} =

b) 3.8×10^3 =

e) 6.4×10^{-4}

c) 6.2×10^0 =

f) 2.9×10^{-1}

TO ADD OR SUBTRACT NUMBERS IN SCIENTIFIC NOTATION, FIRST re-write the numbers so that all are expressed in the same power of ten; then add/subtract the numerical factors and express the sum of difference to the proper power of ten.

EXAMPLE 13

$$9.8 \times 10^5 = 9.80 \times 10^5$$

$$+3.4 \times 10^4 = \underline{\quad .34 \times 10^5 \quad}$$

$$10.14 \times 10^5 = 1.1014 \times 10^6$$

EXAMPLE 14

$$9.8 \times 10^{-5} = 0.98 \times 10^{-4}$$

$$+3.4 \times 10^4 = \frac{3.40 \times 10^{-4}}{4.38 \times 10^{-4}}$$

NOTE THAT MOVING THE DECIMAL POINT ONE PLACE TO THE LEFT INCREASED THE EXPONENT IN BOTH OF THE PREVIOUS EXAMPLES. (REMEMBER: -4 IS GREATER THAN -5).

TO MULTIPLY OR DIVIDE NUMBERS IN SCIENTIFIC NOTATION, multiply or divide the numerical factors and treat the power of ten using the laws of exponents.

EXAMPLE 15

$$\frac{5 \times 10^5}{2 \times 10^3} = \frac{(5)(10^5)}{(2)(10^3)} = \frac{5}{2} \times 10^{5-3} = 2.5 \times 10^2$$

EXAMPLE 16

$$(5 \times 10^5)(2 \times 10^3) = (5)(2)(10^{5+3}) = 10 \times 10^8 = 1 \times 10^9$$

EXERCISES

Perform the indicated operations; express all answers in proper scientific notation.

- 1) $(8 \times 10^6) + (9.6 \times 10^7)$
- 2) $(4.2 \times 10^4) - (4.4 \times 10^3)$
- 3) $(6.02 \times 10^{23})(5 \times 10^2)$
- 4) $(9.3 \times 10^{12}) + (3 \times 10^9)$
- 5) $(.00005)(1,000,000)$
- 6) $100,000 \div .005$

1.1.4.4 PRACTICE PROBLEMS

Express the following using exponential notation:

1) $4 \times 4 \times 4 \times 5 \times 5 =$

3) $\sqrt{13}$

2) $2 \times 2 \times 2 \times 9 \times 9 \times 9 \times 9 =$

4) $\sqrt[5]{24^3}$

Solve the following using your calculator

5) $\sqrt{10}$

8) $(79)^{1/3}$

11) $\sqrt{4^3}$

6) $(12)^3$

9) $(45)^{1/2}$

12) $(4)^{2/3}$

7) $(92)^2$

10) $\sqrt[3]{10}$

Solve the following operations, expressing the answers in exponential notation:

13) $(5)^2 \times (5)^4$

17) $(4)^5 \div (4)^{-4}$

14) $(3)^4 \times (3)^5 \times (2)^4$

18) $(10)^0 \div (10)^{-2}$

15) $(6)^7 \div (6)^3$

19) $(2)^3 \times \frac{1}{(2)^2}$

16) $(2)^4 \times (2)^{-2}$

20) $(8)^{1/2} \times (8)^{1/2}$

Express the following using scientific notation:

21) 37,000,000,000 disintegrations per second

22) .0000064 seconds

23) .000502

24) 3,180,000

Solve the following, expressing answers in proper scientific notation:

25) $(5.2 \times 10^{-5})(2 \times 10^3)$

30) $(5 \times 10^{-8}) - (4 \times 10^{-9})$

26) $(1.5 \times 10^8) + (5 \times 10^4)$

31) $(3 \times 10^{-3}) - (3 \times 10^{-2})$

27) $(8 \times 10^{-2}) + (2 \times 10^{-3})$

32) $(9 \times 10^8) + (7 \times 10^6)$

28) $(7.15 \times 10^4)(4 \times 10^{-1})$

33) $(7.6 \times 10^4) + (4 \times 10^3)$

29) $(6.8 \times 10^5) - (3.4 \times 10^4)$

1.1.5 SYMBOLS OF INCLUSION

For expressions that contain several terms and that require more than one of the fundamental operations to solve, it becomes necessary to use GROUPING SYMBOLS OF INCLUSION. The symbols represent the order in which each operation should be performed to arrive at the intended solution.

Grouping symbols in common use include parentheses (), brackets [] and braces { }. The following examples show the effect grouping symbols have on the intended meaning of an expression.

EXAMPLE 1

To indicate that 5 is to be subtracted from the difference of 15 - 8, the following symbols must be used.

$$(15 - 8) - 5$$

Performing the operations in the indicated order:

$$(15 - 8) - 5 = (7) - 5 = 2$$

To indicate that the difference of 8 - 5 is to be subtracted from 15, the expression would be written:

$$15 - (8 - 5) = 15 - (3) = 12$$

Thus, if grouping symbols were not used or if no rule for the order in which the operations are to be performed is given, and the expression were merely written as 15 - 8 - 5, it would be possible to obtain two different solutions.

GROUPING SYMBOLS ARE USED TO MAKE THE INTENDED MEANING OF THE EXPRESSION CLEAR. In general, PERFORM THE OPERATIONS NECESSARY TO REMOVE THE INNERMOST GROUPING SYMBOLS FIRST; THEN THE NEXT INNERMOST GROUPING SYMBOLS; THEN THE NEXT, etc. UNTIL THE EXPRESSION IS SOLVED.

EXAMPLE 2

Consider the following example:

$$[(2 \cdot 3 + 1) - 7] + 6^2 \div 3 + \sqrt{36}$$

Since the innermost parentheses must be simplified first, a value must be obtained for $2 \cdot 3 + 1$. However, unless we also establish rules for the order in which arithmetic operations are to be performed in such an expression, we could

- a) Multiply 2 by 3 and then add 1 obtaining an answer of 7.
- b) Add 3 and 1, then multiply by 2 obtaining an answer of 8.

The following rules, in the order shown, govern the order in which the operations must be performed:

- 1) Perform all powers in the expression as read from left to right.
(Remember: roots are also powers.)
- 2) Perform all multiplications and/or divisions as read from left to right.
- 3) Perform all additions and/or subtractions as read from left to right.

With the rules above, and the rules governing symbols of inclusions, we are now prepared to find the solution to the example.

$$[(2 \cdot 3 + 1) - 7] + 6^2 \div 3 + \sqrt{36}$$

$$[(6 + 1) - 7] + 6^2 \div 3 + \sqrt{36}$$

(innermost parentheses and rule 2)

$$(7 - 7) + 6^2 \div 3 + \sqrt{36}$$

(innermost parentheses and rule 3)

$$0 + 6^2 \div 3 + \sqrt{36}$$

(remaining parentheses and rule 3)

$$0 + 36 \div 3 + 6$$

(rule 1)

$$0 + 12 + 6$$

(rule 2)

$$18$$

(rule 3)

REMEMBER: Innermost parentheses must be simplified first. To simplify the expression in the parentheses apply rules 1), 2) and 3) IN THAT ORDER.

EXAMPLE 3

Simplify the expression $7 + 4(2)^3 - (18 - 12)$.

a) $7 + 4(2)^3 - 6$

(Simplifying parentheses)

b) $7 + 4 \cdot 8 - 6$

(Raise 2 to the third power)

c) $7 + 32 - 6$

(Perform multiplication)

d) 33

(Add and subtract)

EXAMPLE 4

Simplify the expression $8 + \sqrt[3]{8} - 3(4 - 3)$.

a) $8 + \sqrt[3]{8} - 3 \cdot 1$

(Simplifying parentheses)

b) $8 + 2 - 3 \cdot 1$

(Taking cube root of 8)

c) $8 + 2 - 3$

(Perform multiplication)

d) 7

(Add and subtract)

NOTE: THE RADICAL MAY ALSO BE CONSIDERED AS A GROUPING SYMBOL.

EXAMPLE 5

Simplify the expression $10 + \sqrt[3]{60 + 4} - 4(5 - 6)$

- a) $10 + \sqrt[3]{64} - 4(-1)$ (Simplifying parentheses)
- b) $10 + 4 - 4(-1)$ (Take cube root of 64)
- c) $10 + 4 + 4$ (Perform multiplication observing the LAWS OF SIGNED NUMBERS)
- d) 18 (Final solution)

More on grouping symbols will be discussed in the study of algebra.

EXERCISES

- 1) Perform the indicated operations in the proper order.
- 1) $12 + 5(8 - 3) =$
 - 2) $2[5 + 3(4 - 2)] =$
 - 3) $4 + \sqrt{16} - 3(6 - 4) =$
 - 4) $3[2 + (3)^2] - 2[1 + 3(2 + 4)] =$
 - 5) $[(5)(4) + 3] \cdot 2 + 6 =$
- 2) Group the following expressions to show the intended order of operations.
- 1) The product of 5 and the sum 2 plus 3.
 - 2) The sum of 5 and the product 3 times the quantity $4 - 2$.
 - 3) The difference of the quantity containing the product of 5 and the quantity $15 - 5$ AND the quantity containing the product of 2 times the quantity $3 + 4(6 - 1)$.

PRACTICE PROBLEMS

Perform the indicated operations:

1. $(15 - 10) - 3$

2. $6(4 + 2) - 7$

3. $4(3^2 - 2^2) + 5$

4. $10 - 8 - [6 - (4 - 2)]^2$

5. $5[(9 - 6)^2 - \sqrt{16}]$

6. $7 - \{6 - [5^2 - 4(2 + 3)]\}$

7. $\frac{\frac{32}{8}}{4}$

8. $2\sqrt{5^2 - 4^2}$

Chapter One: SOLUTIONS TO EXERCISES:

Page 3 :

A. 1) 4 2) 10 3) 10 4) 2 5) 35 6) 236

B. 1) 36 2) -36 3) -42 4) -9 5) -20 6) 20

C. 1) 0 2) 20 3) -5 4) 16 5) 2 6) 8

Page 6 :

1) 63 2) 9 3) -24 4) -30 5) -12 6) 241 7) -121

8) 2 9) 11 10) 9 11) -2 12) 30 13) -8 14) 10 15) -4

Page 8 :

1) 0 2) 30 3) -30 4) -36 5) 261 6) 12 7) 50 8) 100

9) 84 10) 54 11) -54 12) -45 13) -4 14) 6 15) 210

Page 10 :

1) 60 2) 40 3) 42 4) 12 5) -150 6) -52 7) -144 8) 128

9) -24 10) -36 11) 0 12) 0

Page 11 :

1) -2 2) -14 3) 9 4) 0 5) 5 6) -7 7) -4 8) 0

9) 6 10) -23 11) 31 12) undefined

Page 16 :

1) $1/6$ 2) $39/64$ 3) $-7/8$ 4) $-15/72$ ($-5/24$) 5) $-10/54$ ($-5/27$)

6) $-6/56$ ($-3/28$) 7) 0

Page 18 :
1) 4/3 2) 16/21 3) 0 4) 4/93 5) -81/64 6) 56/24 (7/3)

Page 19 :
1) 1/3 2) 2/3 3) 5/9 4) 1/3 5) -1/3 6) 1/5

Page 21 :
2) 4/18, 9/18 3) 9/56, 49/56 4) 9/12, 8/12 5) 39/45, 40/45 6) 45/9, 2/9

Page 22 :
1) 3/4 2) 7/6 3) 32/21 4) 13/24 5) -1/10 6) -1/2 7) 19/24
8) 10/33 9) 11/78 10) -5/6 11) -3/8 12) 7/32

Page 23-24 :
1) 4 3/8 2) 3 2/9 3) 4 1/16 4) -2 4/7 5) 49/8 6) 19/2
7) 85/4 8) -46/7 9) 4 1/4 10) 7 71/72 11) 9 5/6 12) 11 3/16
13) 2 1/6 14) 13 1/8 15) 11/28 16) 2 5/8 17) 119/6 (19 5/6)
18) -85/64 (-1 21/64) 19) 5/2 (2 1/2) 20) -9/2 (-4 1/2) 21) undefined

Page 26 :
1) 32 2) 1/54 3) 15/8 4) -12/5 5) 15/4 6) 10/3

Page 30 :
1) 38.196 2) 125.060 3) 68.25 4) 9.82 5) 8.5 6) -66.65

Page 31 :
1) 136.85 2) 108.847 3) 96.561 4) 5.3 5) 5.25 6) 500

Page 36 :

- 1) a) 5 b) 4 c) 3 d) 2 e) 5 f) 2 g) 3 h) 2
2) a) .75 b) .125 c) .3125 d) .28125
3) a) .333 b) 3.27 c) 75.8 d) 19600
4) 62.5% 5) 577.5 6) 21% O, 78% N, 1% Ar

Page 40 :

- TOP: 1) 4^6 2) 6^1 3) 16^3 4) X^4 5) $3^2 \cdot 5^3$
BOTTOM: 1) 8 2) 4.563 3) 2.621 4) 8.246

Page 42 :

- 1) 16^{12} 2) 7^5 3) 5^{10}

Page 43 :

- 1) 6^4 2) 5^6 3) 2^5

Page 45 :

- 1) 3^{28} 2) 2^{25} 3) 5^{22}

Page 46 :

- TOP: 1) $\frac{3^7}{5^7}$ 2) $\frac{2^6}{7^6}$ 3) $\frac{5^8}{9^8}$

- BOTTOM: 1) $\frac{1}{7^5}$ 2) $\frac{1}{3^2}$ 3) $\frac{1}{5}$

Page 48 :

- 1) a) $11^{1/4}$ b) $2^{1/2}$ c) $17^{1/5}$
2) a) $\sqrt[3]{7}$ b) $\sqrt{7}$ c) $\sqrt[4]{2}$
3) a) 2.351 b) 7.211 c) 2.620

Page 49 :

- 1) 16^5 2) 13^3 3) 21^4 4) $1/5^2$ 5) $1/11^3$ 6) $1/2$ 7) 1
8) 1 9) $12^{1/5}$ 10) 3^6 11) 2^2 12) $1/2^6$

Page 51 :

- a) 3.6×10^4 b) 5.14×10^6 c) 6×10^{-6} d) 1.25×10^{-3}
a) 1,500,000 b) 3,800 c) 6.2 d) .001 e) .00064 f) .29

Page 52 :

- 1) 1.04×10^8 2) 3.76×10^4 3) 3.01×10^{26} 4) 3.1×10^3
5) 5×10^1 6) 2×10^7

Page 58 :

- 1) 37 2) 22 3) 2 4) -5 5) 52
1) $5(2 + 3)$ 2) $5 + 3(4 - 2)$ 3) $5(15 - 5) - 2 [3 + 4(6 - 1)]$

Chapter One: SOLUTIONS TO PRACTICE PROBLEMS:

Page 12-13:

- 1) -12 2) -1130 3) -63 4) 114 5) -30 6) 0
7) 48 8) -66 9) 7 10) -220 11) 72 12) 387
13) 6 14) 18 15) 186 16) -240 17) -210 18) 72
19) 0 20) 1000 21) 72 22) -12 23) 0 24) -9
25) 12 26) -8 27) 105 28) positive 29) negative 30) -2/3
31) -4/3 32) -15/4

Page 27-28:

- 1) $\frac{3}{5}$ 2) $\frac{3}{14}$ 3) $\frac{7}{18}$ 4) $\frac{1}{4}$ 5) $1\frac{1}{4}$ 6) $\frac{4}{33}$ 7) $1\frac{7}{8}$ 8) $1\frac{5}{13}$
9) $\frac{38}{45}$ 10) $1\frac{7}{12}$ 11) $\frac{20}{21}$ 12) $1\frac{7}{16}$ 13) $\frac{1}{18}$ 14) $\frac{4}{15}$ 15) $\frac{13}{32}$ 16) $\frac{3}{28}$
17) $1\frac{1}{6}$ 18) $\frac{32}{35}$ 19) $17\frac{5}{6}$ 20) $2\frac{1}{12}$ 21) $\frac{5}{21}$ 22) 0 23) $1\frac{1}{3}$ 24) 4
25) 1 26) $2\frac{8}{9}$ 27) $1\frac{1}{2}$ 28) $\frac{51}{56}$ 29) $\frac{-7}{40}$

Page 37-38:

- 1) 217.8 2) 3.036 3) 3.345 4) 0.003662 5) 205.1
6) 1.02 7) 422.162 8) 0.005931 9) 655.2 10) 18.816
11) 0.06156 12) 0.23738 13) 9.6 14) 300 15) 39.2
16) 0.4637 17) 22,900 18) 199 19) 5.03 20) 425°F
21) 60% 22) 134055 pounds 23) 67.0%

Page 53-54:

- | | | | | |
|---------------------------|--------------------------|---------------------------|------------------------|---------------------|
| 1) $4^{3.5^2}$ | 2) $2^{3.9^4}$ | 3) $13^{1/2}$ | 4) $24^{3/5}$ | 5) 3.162 |
| 6) 1728 | 7) 8464 | 8) 4.291 | 9) 6.708 | 10) 2.154 |
| 11) 8 | 12) 2.520 | 13) 5^6 | 14) $3^9 \cdot 2^4$ | 15) 6^4 |
| 16) 2^2 | 17) 4^9 | 18) 10^2 | 19) 2 | 20) 8 |
| 21) 3.7×10^{10} | 22) 6.4×10^{-6} | 23) 5.02×10^{-4} | 24) 3.18×10^6 | |
| 25) 1.04×10^{-1} | 26) 3×10^3 | 27) 4×10^1 | 28) 2.86×10^4 | |
| 29) 6.46×10^5 | 30) 4.6×10^{-8} | 31) -2.7×10^{-2} | 32) 9.07×10^8 | 33) 8×10^4 |

Page 59:

- 1) 2 2) 29 3) 25 4) -2 5) 25 6) 6 7) 1 8) 6