APPENDIX

February 4, 2009

Volume III – Technical Appendix

DUFF & PHELPS

Table of Contents

APPENDIX

Volume III

A. Overview of Certain Valuation Methodologies

The Black-Scholes-Merton formulas for the valuation of calls and puts are justifiably famous. What is less well-known is the valuation progress that was made after the formula discovery. In particular, people discovered that they could apply numerical methods to extend valuation capacity to include derivatives that were too complex to fit the limiting assumptions used to derive the formula. This report employs two numerical methods, Monte Carlo simulation and lattices.

Monte Carlo simulation was developed by scientists working on the Manhattan Project in the 1940s. The essence of the idea is to repeat a process many times, observe the outcome and use that information to calculate the answer sought. Consider this simple example. A person wants to know what the odds are for each value for the sum of the throw of two dice. This is a relatively simple problem to solve using basic logic. Now, suppose someone wishes to know the probability of getting a sum of 33 on the throw of 10 dice. Simple logic will not suffice and deriving a solution to this problem will be very challenging. It is feasible and relatively simple, however, to have a computer "throw" 10 dice a large number of times and record the outcomes. This produces a Monte Carlo simulation estimate of the true probability of the sum being 33. The larger the number of times the computer throws the dice, the more accurate is the estimate.

Monte Carlo simulation can be used to value derivatives. In the simplest case, you can simulate a common stock price and use the values produced to replicate the results of the Black-Scholes-Merton formula. Of course, the purpose is to use the method to answer more complex questions. This report uses Monte Carlo simulation in two ways. Frist, we value the warrants using volatility and interest rates that vary over time because we think this is a more accurate representation of reality. Monte Carlo simulation also allows us to model conditions under which refinancing would cancel one-half of the warrants. Second, we use Monte Carlo simulation to model the evolution of the value of the assets of the subject companies over time in the Contingent Claims Analysis. The CCA has a significant number of complexities that require simulation.

A lattice of values for the underlying random variable can be used to calculate derivative values. A lattice is akin to a simulation with the exception that in each interval of time the underlying variable can move to only one of two values, given its current value. The price lattice on the next page, which is a representation of the price movements of a common stock over a 6-week period, illustrates this. The stock price starts at 100 and at the end of the first week can be either 102.83 or 97.28. In this model the probability of each is 0.5. If the price moves to 102.83 at the end of the first week, then at the end of the second week it can be either 105.74 or 100.04. The lattice summarizes the distribution of the stock price at the end of 6 weeks in terms of the 7 prices shown with their associated probabilities. For example, the probability of the price being 105.83 is 23.4%. The more steps there are in a lattice, the more accurately it models the movements of the stock price, which obviously can assume more than just 7 prices. Lattices are useful for valuing derivatives when there are call provisions and that is how we use the interest rate lattice discussed in another appendix.

Price Lattice Illustration
Price Movements of a Common Stock Over a 6-week Period

Week 0	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Probability
						118.24%	1.6%
					114.99%		
				111.82%		111.86%	9.4%
			108.74%		108.78%		
		105.74%		105.79%		105.83%	23.4%
	102.83%		102.87%		102.91%		
100.00%		100.04%		100.08%		100.12%	31.3%
	97.28%		97.32%		97.36%		
		94.64%		94.68%		94.71%	23.4%
			92.07%		92.10%		
				89.57%		89.60%	9.4%
					87.13%		
						84.77%	1.6%
						Total	*100%*

B. Interest Rate Call Option Valuation

Many fixed income securities have provisions that give the issuer the right to call the security, that is buy the security from the holder at predetermined price. The issuer's opportunity to call the security is adverse to the interests of the holder of the security: The issuer will call the security when it is worth more than the call price. Consequently, the prices of callable securities are lower and their indicated yields are higher. The extent of the influence varies depending on the terms of the securities. This presents two valuation challenges. The first is to examine the publicly traded securities and use their prices and yields to determine the required rate of return on securities, adjusted for the influence of the call right. The second is to value the subject securities adjusting for the call right. We use the lattice method to accomplish these two tasks.

An example will illustrate the process of analysis and valuation. Assume that we observe a bond in the market place that pays an annual interest rate of 6.50% and has a 5-year life. It is callable at par. Its price in the market is 101.16 per 100 and its yield is 6.10%. Our objective is to determine the yield on the bond, if it were not callable.

We believe the issuer will call the bond if its value, net of accrued interest, is greater than 103. The 3% premium to par is to cover the transaction costs associate with the call. We can construct an interest rate lattice that prices the bond at 101.16 and takes into consideration its call provision and the volatility of interest rates. Following research by Black, Derman and Toy[1], we build the illustrative 5-year interest rate lattice shown below, Lattice A. Lattice B shows the bond's value at each date depending on future spot interest rates. Note the nodes in the lattice, representing dates and interest rates, for which the lattice value is 106.50. These are conditions under which the bond is called.

Lattice C represents the value of the call option. The value of the bond, if it were not callable, 106.49, is shown in Lattice D. The yield on this price is 5.00%. We now use this derived interest rate lattice to value a non-traded callable 5-year bond from the same issuer with a coupon rate of 5.75%. The value of this bond is 99.87 as shown in Lattice E and its yield is 5.80%. The two bonds by the same issuer have different yields because of the call provisions. Lattice F shows the derivation of the value of the call option on this second bond and Lattice G displays the calculation of its value if it were not callable.

In summary, we have used the market price of the traded bond to infer its option adjusted yield, which we then use to value the non-traded bond, capturing the value of the promised payments and of the issuer's right to call.

[1] Black, F., E. Derman and W. Toy. 1990 "A one-factor model of interest rates and its application to Treasury bond options," *Financial Analysts Journal*, January/February 33 – 39.

Lattice A
Illustrative 5-year Interest Rate Lattice

Year	0	1	2	3	4	5
					10.19%	
				8.45%		
			7.02%		6.83%	
		5.85%		5.66%		
	4.88%		4.71%		4.58%	
		3.92%		3.80%		
			3.16%		3.07%	
				2.54%		
					2.06%	

Lattice B
Bond Value at Each Date Based on Future Spot Interest Rates

Year	0	1	2	3	4	5
					102.68	106.50
				102.37		
			104.42		105.97	106.50
		107.00		107.70		
	101.66		108.68		108.23	106.50
		106.50		106.50		
			106.50		106.50	106.50
				106.50		
					106.50	106.50

Lattice C
Value of the Call Option

Year	0	1	2	3	4	5
					0.00	0.00
				0.00		
			0.00		0.00	0.00
		1.11		0.00		
	4.83		2.36		0.00	0.00
		9.03		4.95		
			9.24		3.28	0.00
				7.53		
					4.33	0.00

Lattice D
Value of the Bond

Year	0	1	2	3	4	5
					102.68	106.50
				102.37		
			104.42		105.97	106.50
		108.11		107.70		
	106.49		111.04		108.23	106.50
		115.53		111.45		
			115.74		109.78	106.50
				114.03		
					110.83	106.50

Lattice A
Illustrative 5-year Interest Rate Lattice

Year	0	1	2	3	4	5
					10.19%	
				8.45%		
			7.02%		6.83%	
		5.85%		5.66%		
	4.88%		4.71%		4.58%	
		3.92%		3.80%		
			3.16%		3.07%	
				2.54%		
					2.06%	

Lattice E

Year	0	1	2	3	4	5
					101.26	105.75
				100.30		
			101.71		104.52	105.75
		103.97		105.58		
	99.87		106.55		106.77	105.75
		105.75		105.75		
			108.22		108.30	105.75
				105.75		
					105.75	105.75

Lattice F

Year	0	1	2	3	4	5
					0.00	0.00
				0.00		
			0.00		0.00	0.00
		0.79		0.00		
	3.38		1.68		0.00	0.00
		6.31		3.53		
			4.66		0.00	0.00
				6.09		
					3.60	0.00

Lattice G

Year	0	1	2	3	4	5
					101.26	105.75
				100.30		
			101.71		104.52	105.75
		104.76		105.58		
	103.25		108.24		106.77	105.75
		112.06		109.28		
			112.88		108.30	105.75
				111.84		
					109.35	105.75

C. Credit Default Swap Valuation

This approach is a discounted cash flow analysis in which the Fair Market Value of the TARP Preferred Stock is estimated as the present value of its adjusted cash flows discounted at the risk free rate. We implement this valuation by estimating the present value of adjusted cash flow from CDS rates for the debt of each firm.

Credit Default Swap and Default/Survival Probabilities

Credit default swaps are contracts in which the buyer makes periodic payments (premium) to the seller. In return, the seller pays an amount that makes the buyer whole (protection) if the underlying instrument defaults. The premium is usually expressed as a "spread", a percentage of the notional amount of the underlying instrument. Because the level of premium the CDS seller demands is a function of the risk of the underlying instrument, adjustments that incorporate both default probabilities and risk premiums can be inferred from the level of CDS spreads observed in the market. Because these adjustments[2] include the expected loss from default and the required risk premium, the adjusted cash flows are discounted at the risk-free rate of interest.

In a CDS contract, the expected premium payments can be expressed as:

Premium Leg = $$S_N \sum_{i=1}^{N} DF_i \cdot PND_i \cdot \Delta_i + S_N \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i) \cdot \frac{\Delta_i}{2}$$

where:

S_N is the Par Spread (Annual Swap Rate) for maturity N,
DF_i is the Riskless Discount Factor from T_0 to T_i,
PND_i is the Survival Probability from T_0 to T_i,
Δ_i is the Accrual Period from T_{i-1} to T_i,

and the expected protection (or "contingent") payment can be expressed as:

Contingent Leg = $$(1-R)\sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)$$

where:

R is the recovery rate,
PND_i is the Survival Probability from T_0 to T_i,
PND_{i-1} is the Survival Probability from T_0 to T_{i-1},

An efficient construct for modeling probabilities of default is the concept of Hazard Rates. A hazard rate h (t) denotes the probability of default at time 't', conditional on survival till time 't'. That is,

$$h(t) = \frac{pd(t)}{1-PD(t)} = \frac{pd(t)}{PND(t)}$$

[2] For convenience, we will refer to the adjustments as probabilities of default and their complements as survival probabilities. This convention follows from the fact that the values of the adjustments fall between 0.0 and 1.0 and play a computational role similar to that played by true probabilities.

Since h (t) is also equal to the negative of the derivative of ln {PND (t)}, therefore

$$PND(T) = e^{-\int_{o}^{T} h(t)dt}$$

We assume a piecewise-constant hazard function h(t), with one "piece" corresponding to each section of maturity horizon for which we have par spreads available. This is consistent with common industry practice.

The Fair Market Value of a CDS contract can be calculated as the Premium Leg less the Contingent Leg. In an arms-length transaction, the Fair Market Value is zero as of the transaction Date. Stated differently, on the transaction date, the Premium Leg should equal the Contingent Leg. Therefore, we can solve for the hazard rate (h) implied by the market CDS spreads observed on the valuation date by solving for the hazard rate that equates the two Premium Legs to the Contingent Leg.

The CDS spread levels that we use were obtained from Fitch CDS service. These levels represent consensus "end-of-day" quotes received from broker/dealers. Depending on the number of market makers actively providing quotes for a given name, such consensus takes into account information obtained from a minimum of 9 and up to 22 different broker/dealers. We use the CDS spreads for senior unsecured debt, as this is usually the most liquid contract. We use a full term structure of liquid spreads that are available for different maturities. The observed CDS spreads for each of the Purchase Program Participants are presented below:

Observed CDS Spreads

Term(Yrs)	Goldman	BofA	JPMorgan	Morgan Stanley	Citigroup - CPP	Wells Fargo	PNC	USB	AIG	Citigroup - TIP
	10/14/2008	*10/14/2008*	*10/14/2008*	*10/14/2008*	*10/14/2008*	*10/14/2008*	*10/24/2008*	*11/3/2008*	*11/10/2008*	*11/24/2008*
1	414	57	57	666	192	27	75	86	1026	264
2	271	69	64	562	186	36	n/a1	91	1133	249
5	201	105	88	423	181	39	n/a1	108	1168	243
7	178	107	89	383	183	70	n/a1	114	1062	232
10	168	100	86	336	181	70	n/a1	114	976	227

Sources: Capital IQ, Bloomberg.

Cash Flow Analysis

For a given period, the expected (i.e. probability adjusted) cash flow for the holder of the TARP Preferred Stock can be calculated as the sum of: (1) the required coupon payment multiplied by the survival probability for that period; (2) the recovery amount if default occurs multiplied by the default probability for that period; and (3) the face value if redemption is assumed to occur.

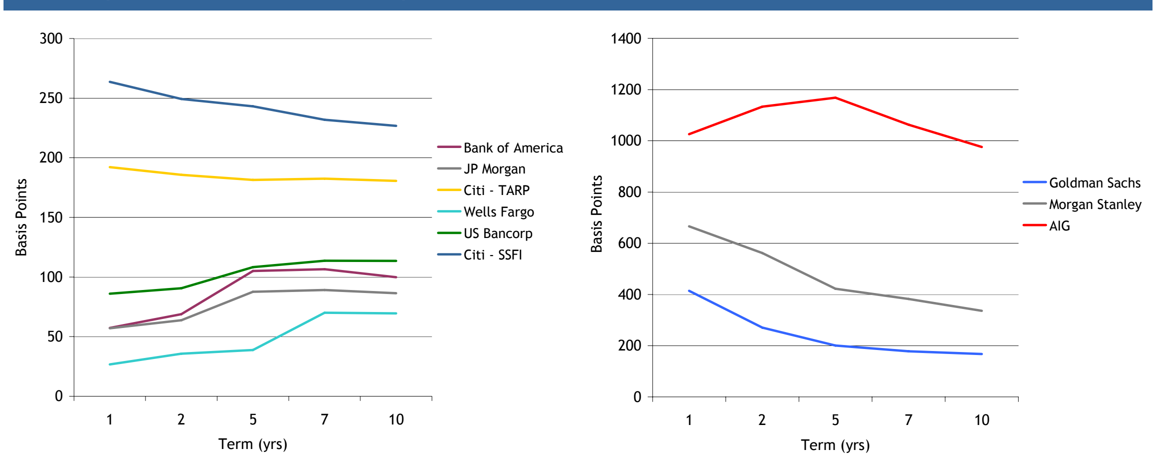
Recovery Rate

For purposes of our analysis, we assumed the recovery rate upon default (referenced in (2) above) to be zero. We believe this is a reasonable assumption because recent bankruptcy data for financial institutions suggests the recovery rate for debt instruments is generally in the 9% range. Given that preferred stock has lower seniority than debt, we expect the TARP Preferred Stock's recovery rate to be significantly lower – perhaps in the 0% to 2% range. Furthermore, our sensitivity analysis indicated that our value conclusion is not significantly impacted by the recovery rate assumption when the recovery rate is expected to be in the 0% to 10% range. Therefore, item (2) in the previous paragraph was eliminated from our cash flow formula.

Redemption Assumption

We understand that the Purchase Program Participants can redeem the TARP Preferred Stock upon a qualified equity offering or any time after the third anniversary. The actual redemption date is a function of Purchase Program Participant's ability to raise financing at a lower cost. Since the TARP Preferred Stocks are not actively traded in the marketplace, it is not possible to directly infer market's expectation regarding the timing of redemption. Thus, judgment is required for this input. We believe those Purchase Program Participants with a lower cost of capital are likely to redeem the TARP Preferred Stock earlier. Also, since the dividend in the CPP will reset from 5% to 9% in five years, the incentive for redemption would be stronger after year five. Assuming redemption is expected when the cost of capital falls, some observations can be drawn from the shape of the par CDS curves. Almost all of the Purchase Program Participants have CDS curves that are inverted, after peaking between a two- to five-year time horizon. This implies the CDS market's view that cost of funding in the medium term future can be expected to be lower. Examining at this data, it can be argued that expected redemption for such institutions is between three and five years.

CDS Curves (drawn twice for differences in scale)



Discount Rate

Because the probability of default is accounted for in the derivation of the cash flows, the appropriate discount rate is the risk free rate. We derived the appropriate risk free rate from the CMT Rates. The follow table presents the risk free rates we utilized for our analysis:

Historical Constant Maturity Treasury (CMT) Rates

Term (Years)	Goldman, BofA, JPMorgan, Morgan Stanley, Citigroup-CPP, Wells Fargo	PNC	USB	AIG	Citigroup - TIP
	10/14/2008	*10/24/2008*	*11/3/2008*	*11/10/2008*	*11/24/2008*
0.25	0.33	0.84	0.44	0.2	0.01
0.5	0.92	1.41	0.98	0.81	0.45
1	1.22	1.56	1.28	1.14	0.92
2	1.82	1.51	1.44	1.24	1.2
5	3.02	2.58	2.7	2.49	2.2
10	4.08	3.69	3.91	3.74	3.32
30	4.28	4.07	4.32	4.19	3.78

Source: Bloomberg.

Conclusions

In general, for the stronger banks the spread increases are in the 100 to 250 basis points range. This level of adjustment captures reasonable value ranges for all cases except Morgan Stanley, the second Citigroup investment and AIG, the lower value investments where the risk exposure of preferred stock holders may be heightened relative to the other banks.

Comparison of Yield-Based DCF Values to CDS-Based DCF Values

	Yield-Based DCF Values			Values Computed Using CDS Rates Plus Additional bps Spreads of:					
Company	**High Yield**		**Low Yield**	**bps Increase**		**bps Increase**	**Value**		**Value**
AIG	$15.7	to	$16.5	300	to	400	$17.0	to	$18.3
BofA	$12.2	to	$13.2	150	to	250	$11.3	to	$14.0
Citigroup-CPP	$14.8	to	$16.1	150	to	250	$14.9	to	$18.1
Citigroup-TIP	$8.9	to	$9.4	450	to	550	$8.7	to	$10.5
Goldman	$6.7	to	$7.2	150	to	250	$6.7	to	$8.2
JPMorgan	$20.8	to	$22.3	150	to	250	$19.0	to	$23.5
Morgan Stanley	$4.8	to	$5.2	250	to	350	$4.9	to	$5.7
PNC	$5.4	to	$5.8	*Insufficient CDS Rate Data*					
USB	$6.1	to	$6.6	100	to	200	$5.3	to	$6.6
Wells Fargo	$22.0	to	$23.8	100	to	200	$19.3	to	$25.1

Note: $s in billions.

D. Contingent Claims Analysis Valuation

We implemented contingent claims analysis as described in Lucas and McDonald (2005, 2008). The key assumptions are outlined below.

Evolution of Assets

Let A denote the value of each Purchase Program Participant's assets. We assume that the rate of return on assets is normally distributed and implement a risk-neutral discrete time evolution of assets in the Monte-Carlo simulation as:

$$A_{t+h} = A_t Exp\left[\left(r_f + g_t - \delta \frac{E_0}{A_0} - 0.5\,\sigma_A^2\right)h + \sigma_A e\sqrt{h}\right]$$

where:

h is the time step,
E is equity,
r_f is the risk-free rate,
g_t is the externally financed firm asset growth,
d is the dividend yield,
σ_A is the volatility of firm assets, and
e is a draw from a standard normal distribution.

Asset Volatility

In the standard approach you estimate the initial market value and volatility of assets based on treating equity, E, as a call option on the underlying assets of the firm, according to:

$$E = Ae^{-qT}N(d_1) - Xe^{-r_f T}N(d_2) + A(1-e^{-qT})$$

$$\sigma_A = \sigma_E \frac{E}{A}[N(d_1)e^{-qT} + (1-e^{-qT})]^{-1}$$

$$d_1 = [\ln(A/X) + (r - q + .5\sigma_A{}^2)T]/(\sigma_A T^{.5})$$

$$d_2 = d_1 - \sigma_A T^{.5}$$

where:

T is the maturity of liabilities,
X is the strike price,
σ_E is equity volatility
σ_A is asset volatility
$q=\delta^* E_0/A_0$ is the payout rate of assets, and
all values are as of time 0.

The equation relates σ_E and σ_A as follows: $\sigma_E = (\partial E/\partial A)^*(A/E)^*\ \sigma_A$.

DUFF & PHELPS

Given the market value of equity and the other parameters, you can simultaneously solve for asset value, *A* and asset volatility, σ_A.

We started following the usual approach. The major challenge here was the estimate of equity volatility for the 10-year time horizon. The rapidly changing market values and equity volatility combined with clear evidence of expectations that equity volatility would decrease substantially over time equity volatility created major problems in using the standard approach. After considerable examination of the data, we concluded that we would produce more reliable estimates of value if we determined asset values and volatilities separately. Therefore, we measured the value of assets as the book value of debt and preferred stock and the market value of common stock. Our analysis of the data suggested that the volatility of the depository banks is in the range of 7.0% to 5.0%, with the assets of Goldman Sachs and Morgan Stanley being somewhat riskier and those of AIG, riskier still For the least risky group we employed 1) an asset volatility of 7.0% decreasing linearly over time to 6.0% and 2) an asset volatility of 6.0% decreasing linearly over time to 5.0%. For Goldman Sachs and Morgan Stanley, we added 0.5% to each value producing 7.5% to 6.5% and 6.5% to 5.5%. For AIG, which has even riskier assets we added an additional 0.5% and valued AIG’s Preferred Stock for asset volatilities of 8.0% to 7.0% and 7.0% to 6.0%.

Debt Policy

Let *L* denote the liabilities of each Purchase Program Participant. Following Lucas and McDonald (2008), we assume that the book value of liabilities adjusts towards a target liability to asset ratio at several different adjustment rates. The evolution of liabilities is modeled as:

$$L_{t+h} = L_t e^{(r_d + \gamma g_t)h} + I_t \alpha_t h \left[\lambda * - L_t e^{r_d h} / A_t\right] A_t$$

where:

- α is the annual rate of adjustment,
- l^* is the target liability to asset ratio,
- I_t is an indicator variable that equals one in a period where liabilities are adjusted and 0 otherwise,
- r_d is the growth rate of liabilities to cover promised coupons.
- g represents the fraction of externally financed growth supported by debt.

Ideally, debt is managed to maximize the value of equity especially for highly leveraged financial institutions. This assumption allows us to include the effect of the dynamics of the Purchase Program Participant’s liabilities on equity value.

Conditions for Default

Default risk is the uncertainty surrounding a firm’s ability to service its debts and obligations. We check for default periodically (monthly) and default is triggered when the ratio of asset to liabilities falls below a threshold. We explored the effects of various choices and selected a 0.9 trigger value.

Asset Distribution Rules

To examine the sensitivity of the value of the Preferred Stock to different exit strategies of each Purchase Program Participant, we assume that the asset value will be allocated among each security based on different asset distribution rules. These rules specify each security's payoff breakpoints, which reflect the transition points between the payoffs of the securities and serve as exercise prices in the call option valuation.

We implemented three distribution rules:

1. Strict seniority in which debt receives 100% of asset value in the event of default;
2. Sharing between Debt and Preferred; and
3. Sharing among Debt, Preferred Stock and Common Stock

We adopted a sharing rule whereby preferred stock receives 0% of face value in the event of default. The results are not very sensitive to this assumption in a range around 0% to 5%.

E. Warrant Valuation

We estimated each Purchase Program Participant's volatility in three ways: i) we measured volatility based on historical changes in the Purchase Program Participant's stock prices, Historical Volatilities; ii) we developed forecasts of volatility based on historical prices and assuming a generalized autoregressive heteroskedastic (GARCH) model of price behavior, GARCH Volatilities; iii) we identified volatilities implied by the prices of options, Implied Volatilities.

i. Historical Volatilities

We estimated the historical equity volatility for each Purchase Program Participant as of its Valuation Date. We did so for the previous 5 years, the 5 years prior to that and for the entire 10-year period. We measured the rates of return over 1-day, 1-week and 1-month intervals. We used the equation below to calculate the historical volatility.

The continuously compounded rate of return per period is r_i, $\bar{r}$ is the mean rate of return, n is the number of observations and M converts the periodic estimate of volatility to an annual volatility. M is 252 for daily observations, 52 for weekly observations and 12 for monthly observations.

$$\sigma = \sqrt{M} * \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(r_i - \bar{r})^2}$$

The table on the next page summarizes the results. The basic assumption underlying these types of estimations of volatility is that the volatility is constant over time and that the rates of return are independent, identically distributed. If that were true then the data from different sample periods and measurement intervals would yield approximately the same estimates of volatility. As it is, there are considerable ranges of estimates for each firm. The clearest pattern appears to be that the longer the measurement interval, that is monthly versus daily and weekly, the lower the estimate of volatility. As these are long-term investments, using estimates based on monthly data would appear most reasonable. We will also report the effects of a range of volatilities on the value of the securities.

Historical Volatilities

Date Range	Company	Daily	Weekly	Monthly	Company	Daily	Weekly	Monthly
1998 to 2008	AIG	60%	58%	66%	JPMorgan	40%	39%	34%
2003 to 2008	AIG	87%	78%	99%	JPMorgan	41%	29%	26%
1998 to 2004	AIG	32%	38%	25%	JPMorgan	42%	45%	39%
1998 to 2008	BofA	36%	39%	28%	Morgan Stanley	46%	48%	39%
2003 to 2008	BofA	42%	29%	29%	Morgan Stanley	48%	37%	39%
1998 to 2004	BofA	33%	45%	27%	Morgan Stanley	47%	55%	39%
1998 to 2008	Citigroup CPP	38%	37%	29%	PNC	31%	29%	25%
2003 to 2008	Citigroup CPP	42%	32%	29%	PNC	35%	25%	21%
1998 to 2004	Citigroup CPP	36%	40%	29%	PNC	32%	31%	28%
1998 to 2008	Citigroup TIP	39%	38%	29%	USB	34%	34%	30%
2003 to 2008	Citigroup TIP	47%	36%	34%	USB	34%	21%	19%
1998 to 2004	Citigroup TIP	36%	39%	29%	USB	38%	40%	36%
1998 to 2008	Goldman	41%	37%	38%	Wells Fargo	32%	36%	24%
2003 to 2008	Goldman	40%	30%	34%	Wells Fargo	37%	44%	23%
1998 to 2004	Goldman	41%	38%	37%	Wells Fargo	28%	28%	25%

ii. GARCH Estimates of Volatilities

In order to examine the impact of recent shocks on the conditional forecast of volatility we calculated a GARCH (1,1) model over 5- and 10-year periods using monthly data. We calculated both the unconditional forecast of volatility and the conditional forecast over a 10-year horizon.

Under GARCH(1,1) the conditional variance, h_t, is a function of both the return from the last period, r_{t-1}, and the conditional variance from last period, h_{t-1}.

GARCH (1,1) $$h_t = \alpha_0 + \alpha_1 * r_{t-1}^2 + \beta * h_{t-1}$$

We carefully explored the potential for improving the estimates of volatility using the GARCH analysis, but given the recent high levels of volatility, the GARCH results did not enhance the estimates and thus are not included in the study.

iii. Implied Volatilities

A forward looking measure often interpreted as incorporating the market's perspective on the price of risk is the implied volatility: Option prices in the market and the underlying constituents of the Black-Scholes-Merton model are used to determine the volatility consistent with or implied by the formula and the prices of the options. We have identified implied volatilities for maturities of 3 months, 6 months, 1 year and 2 years. The results appear in the table below.

Implied Volatility

	3-month Volatility	6-month Volatility	1-year Volatility	2-year Volatility
AIG	162.3%	147.8%	142.7%	128.2%
BofA	72.8%	65.4%	59.8%	48.7%
Citigroup	71.9%	63.1%	56.7%	43.9%
Goldman	63.0%	56.3%	52.2%	47.6%
JPMorgan	60.3%	55.6%	51.1%	42.3%
Morgan Stanley	117.2%	100.0%	89.0%	80.3%
PNC	60.2%	55.7%	52.0%	47.7%
USB	57.0%	52.8%	46.9%	35.2%
Wells Fargo	65.3%	56.1%	51.6%	47.8%

Source: Bloomberg

Conclusion

Based on these data and the average historic volatility, we calculated estimates of future monthly volatilities that were close to the implied volatilities, declined smoothly over time and had an average value equal to the historic average. We illustrate this process for BofA.

Time-Vary Volatility for Bank of America