Investing the Social Security Trust Fund in Equities: An Option Pricing Approach

by

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* Macroeconomics Analysis Division, 2nd and D Streets, S.W., Washington, DC 20515. Email: kents@cbo.gov; Phone: (202) 226-2758; Fax: (202) 226-0332. A December, 1996 and a later draft of this paper circulated under the name "Investing the Social Security Trust Fund in Equities: Unmasking the Large Hidden Actuarial Tax Liability on Future Generations." I am very grateful to Peter Diamond and Angelo Mascaro for their extensive comments on several previous drafts of this paper. Helpful comments and advice were also received from Coleman Bazelon, Zvi Bodie, Henning Bohn, Len Burman, Eric Engen, Jane Gravelle, Douglas Hamilton, Andrew Lyon, Olivia Mitchell and Jack Vanderhei. Comments from participants at the April 2 Washington DC Tax Forum and the May 12-13 Social Security Conference at Wharton Business School were also very appreciated. The current draft was prepared for presentation at the 1997 NBER Summer Institute. The views expressed herein do not necessarily reflect those of the CBO.

Abstract

This paper analyzes the three basic allocations of risk among generations from investing the Social Security trust fund in equities using an option pricing approach. That approach produces surprisingly sharp and robust equivalency relationships. These equivalency relationships identify identical fiscal policies—thereby unmasking the intergenerational redistribution of various risk allocations—and also enable a precise calculation of their value.

It is shown that the defined-contribution (DC) risk allocation—in which there is no intergenerational risk shifting—is consistent with the commonly-stated "shell game" scenario that involves a swap between private and public pensions with no price effects.

The current-law defined-benefit (DB) approach, and the approach being implied by some proposals to invest the trust fund in equities, uses the payroll tax to smooth equity gains or losses across generations in order to fix the benefit level. This is mathematically equivalent to an intergenerational transfer on a pay-as-you-go basis of put and call stock options (i.e., a futures contract) that insures that the equity portion of the trust fund performs as expected. This creates an instant windfall for current workers, contracts the expected-value budget constraints for all future workers, and reduces national saving relative to a baseline policy of maintaining the current payroll tax rate. Most importantly, the DB allocation places an actuarial tax liability on future workers that is equivalent to keeping the trust fund invested in bonds and instead raising the same expected revenue by increasing the payroll tax on *only* future workers—a result that holds for any value of risk aversion. Indeed, future generations would be better off if the explicit payroll tax is increased immediately so that current workers participated in maintaining their own future level of benefits. This equivalency result is ironic given the strong political resistance to increasing the payroll tax under the current-law DB system, especially by those advocating trust fund investment in equities as an alternative.

In the asymmetric (ASY) risk allocation, trust-fund equity returns below expectation are buffered with an increase in the payroll tax while returns in excess of the payroll tax are met with an increase in benefits (rather than a reduction in taxes as in the DB approach). This is a more extreme form of the DB approach and is mathematically equivalent to an intergenerational transfer on a pay-as-you-go basis of only put options that insure that the equity portion of the trust fund performs *at least* as expected. The ASY plan is priced and compared with the DB plan.

The paper then discusses how the results might change if markets are incomplete. It is argued that the complete market analysis underlying the option pricing approach might, under certain conditions, overstate the actuarial costs associated with these three risk allocations but that there are also very compelling reasons why the complete-market analysis might understate the true costs. Suggestions for future research are given.

Key words: Social Security Trust Fund, Intergenerational Risk Allocation, Saving.
JEL Codes: D10, G10, G11, E60, E20

I. Introduction

The almost 6% historic equity premium creates a tempting opportunity to rescue the currently insolvent Social Security system—without raising the payroll tax—by investing the large and growing Social Security trust fund in equities instead of government debt, the asset the trust fund currently holds. This type of plan has been discussed for years but it has taken on a new popularity with the recent 1997 release of the report by the Advisory Council on Social Security.[1]

The merits of investing the trust fund in equities is an open issue. Opponents of investing the trust fund in equities argue that investing the trust fund in equities would be a pure shell game—a mere swap in equity holdings between private and public pensions—with possible adverse price effects. Proponents respond that investing the trust fund in equities could be used as a means to maintain the current level of benefits for current workers without increasing the payroll tax in the future. It is also argued that trust fund investment could be part of a plan to boost national saving.

The key question relating to investing the trust fund in equities is, who will bear the associated risk? This issue was considered in broad terms by the Technical Panel of the recent Advisory Council who wrote:

> A different way to think about the proposal is to realize that the risks and returns in the Social Security agency's [equity] portfolio would be passed on to households, perhaps with some lag. When portfolio returns are high, this improves Social Security benefits or reduces Social Security taxes in the future; when returns are low, this approach reduces Social Security benefits or increases taxes in the future. ... [Another] possibility is that trust fund risk will fall on as yet unborn generations, who have no opportunity to hold stocks now on their own accounts. Holding equities in the fund could give future generations exposure to current stock returns, thus

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[1] The CBO addressed the issue in a 1994 paper titled "Implications of Revising Social Security's Investment Policies." Investing the trust fund in equities has been advocated most recently by 6 members of the 13-member Advisory Council (see *Findings, Recommendations, and Statements* of the Social Security Advisory Council [pp. 25 - 27]). The issue has also been addressed most recently by Stephen Zeldes (1995), Barry Bosworth (1996), Edward Gramlich (1996), Edith U. Fierst (1996), Alan Greenspan (1996), P. Brett Hammond and Mark J. Warshawsky (1996), Lawrence White (1996), the *1997 Economic Report of the President*, Henning Bohn (1997*a*,*b*) and Warshawsky (1997).

improving risk sharing across generations.[2]

There are basically three different possible intergenerational risk allocation combinations associated with investing the trust fund in equities. The paper proceeds from the simplest allocation and moves toward greater complexity. Table 1 summarizes the three risk allocations using a payoff matrix. Descriptions of the three allocations are as follows:

- *The Defined Contribution (DC) Approach — Current workers get the equity return next period, whether large or small*:

  Current workers receive retirement benefits larger than those compatible with current tax rates if the equity fund outperforms the nominally risk-free rate (the rate the fund is currently obtaining) *but* current workers also fully absorb any shortfall in the trust fund in the form of lower future benefits.

- *The Asymmetric (ASY) Approach — Current workers get the equity return next period, guaranteed by future workers to equal at least the expected return*:

  Current workers are rewarded in terms of future retirement benefits that are larger than those compatible with current tax rates if the equity fund performs as expected *but* future workers are pre-committed to fully absorb any shortfall in the trust fund below its expected value in the form of larger taxes. Any return at, or in excess of, its expected value is passed on to current workers in the form of larger benefits relative to those compatible with current tax rates.

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[2] *Technical Panel on Trends and Issues in Retirement Saving*, p. 86.

- *The Defined Benefit (DB) Approach — Current workers get the equity return next period, guaranteed by future workers to exactly equal the expected return.*[3]

  Same as ASY except that any return in the equity's fund in *excess* of its expected value accrues to future taxpayers in the form of lower taxes instead of being passed to current workers in the form of larger future benefits.

Whereas the DC risk allocation does not attempt to smooth the equity returns of the trust fund over time, the ASY and DB risk allocations do. The ASY risk allocation is clearly the most generous to current workers, although some might think that this scheme is too extreme in that it gives current workers all of the "upside" if the equity portion of the trust fund performs well but none of the "downside" if the fund performs below expectations. However, it would be naive to believe that the ASY risk allocation could not be a possible outcome from investing the trust fund in equities since Social Security beneficiaries and those near retirement will compose a very large share of the voting population during the next few decades. The DB risk allocation is probably closer to what the promoters of investing the trust fund in equities would claim that they have in mind because it is consistent with the current legal structure of Social Security in which benefits are paid out independently of short-term fluctuations in economic and demographic variables:

> We reiterate that except for this change in investment policy, Social Security's principles and structure would remain unchanged under this approach. Social Security continues as a defined-benefit plan, with the amount of benefits and the conditions under which they are paid still determined by law rather than by individual

[3] An alternative payoff to the DB plan would be to distribute equity returns in excess of [less than] the *risk-free* rate to future workers in the form of a lower [higher] payroll tax. This allocation changes the risk shifting nature of Social Security which might lead to gains or losses (see below) but this allocation has no impact in a complete market setting. This risk allocation, however, does not meet the objective of avoiding an increase in the payroll tax in order to maintain benefits for current workers since current workers would get paid the risk-free rate (what they are already receiving from the trust fund) regardless of the trust fund's performance. So, for example, this plan is not consistent with the plan advocated by six members of the recent 13-member Advisory Council.

> investments. .... there would be ups and downs in returns but only very long-range trends would matter. And the assumed rate of return, while important, would be secondary to the fact that benefits would remain defined by law rather than by the relative uncertainty of individual investment decisions.[4]

Section II analyzes each risk allocation inside of a very ideal setting with fully informed agents, no transaction costs and complete markets. The key results are as follows:

- The defined-contribution (DC) risk allocation is consistent with the commonly-stated "shell game" scenario that involves a swap between private and public pensions with no change in aggregate saving and no price effects.
- The defined-benefit (DB) allocation is mathematically equivalent to an intergenerational transfer on a pay-as-you-go basis of put and call stock options (i.e., a futures contract) that insures that the equity portion of the trust fund performs as expected. The DB allocation represents an instant windfall for current workers along with a contraction of the expected value budget constraint for all future workers. National saving is lower under the DB scheme relative to a *baseline policy* of maintaining the current payroll tax rate. Probably most important, however, is that the DB allocation places an actuarial tax liability on future workers that is as large as keeping the trust fund invested in bonds and instead raising the same expected revenue by increasing the payroll tax on *only* future workers—a result that holds at any value of risk aversion. Indeed, future generations would be better off if the explicit payroll tax were increased immediately so that current workers participated in maintaining their own future level of benefits. This result is ironic given the strong political resistance to increasing the payroll tax under the current-law DB system, especially by those

[4] See, e.g., *Findings, Recommendations, and Statements* (p. 86) of the recent 1994-1995 Advisory Council on Social Security.

who advocate investing the trust fund into equities.[5]

- The <u>asymmetric</u> (ASY) allocation is a more extreme form of the DB approach and is mathematically equivalent to an intergenerational transfer on a pay-as-you-go basis of only put options that insure that the equity portion of the trust fund performs *at least* as expected. The results are qualitatively similar to the DB plan but quantitatively are slightly stronger.

The key simplifying assumption made in Section II is that markets are complete. This includes the existence of an intergenerational futures market that allows current generations to negotiate futures contracts with future generations via the government; alternatively, current government intergenerational tax policy is such that, if such a market were established, there would be no trades. Assuming complete markets therefore rules out a critical way that fiscal policy can be used to enhance welfare: completing a missing intergenerational futures market. Important recent work on this topic has been done by Douglas Gale (1990) and Henning Bohn (1997*a,b*). Bohn (1997*a*), in particular, provides a comprehensive examination of the potency of various policy instruments in distributing risk between generations in a general-equilibrium framework.

The assumption of complete markets, while certainly unrealistic, facilitates converting actuarial tax liabilities into parsimonious option pricing formulae, which has the advantage of producing surprisingly sharp and robust equivalency relationships within a simple framework. These include the equivalence of the ASY and DB plans and the intergenerational transfer of specific financial instruments. Another, and the most important, equivalence result is that between the DB plan and an explicit tax increase on only future workers that raises the same expected revenue.

[5] While six of the 13 members of the recent Advisory Council advocated investing the trust fund in equities, all members rejected an increase in the payroll tax.

These equivalency relationships not only identify identical fiscal policies—thereby unmasking the intergenerational impact of various risk allocations—but also allow for computing a precise measure of their value.

A more traditional reason for solving the complete market problem reflects our ignorance of just how incomplete a given market really is. It is not true that intergenerational risk shifting is non-existent. The government already shifts risk between generations via safe debt and the pay-as-you-go portion of Social Security, Medicare and by taxing capital income (Gale [1990]; Bohn [1997*a*]). Passing on more risk—even, hypothetically, uncorrelated risk—might therefore be detrimental to future workers. As Bohn (1997*b*, pp. 19-20) writes, "It would take a quite ambitious empirical study to determine whether the old are currently too little or too much exposed to productivity risk, nothing less than a comprehensive survey of all relevant sources of risk affecting U.S. households of different ages." To the extent that investment into the trust fund is viewed as an *increment* to existing policy, the complete market analysis therefore might not be far from the truth. It may even yield conservative predictions: indeed, future generations might be better off under the explicit tax increase than the DB plan that exposes them to additional risk. But, as Bohn (1997*a*) shows, there still might be room for improvement in risk sharing if the government alters its choice of policy instruments altogether.

Finally, complete market analysis helps untangle the details which is important since trust fund investment is not necessarily the only way of completing an imperfect intergenerational futures market.[6] At a minimum, the equivalency of the DB plan and the explicit tax increase under complete markets yields the pungent result that the gains to future generations resulting from a lack of

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[6] I thank Zvi Bodie for this point.

exposure to contemporaneous shocks must be large enough such that future generations would be willing to pay an additional payroll tax—equal to the value that would generate the same expected revenue as investing the trust fund into equities—in order to be exposed to contemporaneous shocks.

Section III describes how the key results derived in Section II change upon relaxing each assumption. Section IV concludes and provides suggestions for future research.

II. Complete Markets

This section makes the following assumptions. Each assumption is discussed in Section III.

ASSUMPTIONS

- A1. *Agents are Fully Informed.*
- A2. *There are no Transaction Costs.*
  - A2.1 *No Trading Costs.*
  - A2.2 *No Capital Gains Taxes.*
- A3. *Agents Face Complete Markets*
  - A3.1 *No Borrowing Restrictions*
  - A3.2 *No Short-Sale Restrictions*
    (Alternatively, agents currently hold equity.)
  - A3.3 *Intergenerational Futures Market is Active*
    (Alternatively, future agents already face optimal tax rate uncertainty.)
- A4. *The Trust Fund is Perpetual*

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1. The Defined Contribution (DC) Allocation: The Shell Game

To see how investing in equities can constitute a pure shell game, consider, without loss in generality, the dynamic budget constraint faced by an agent in a two-period model. The agent, born in time t, makes a labor income equal to Y_1 at age 1 known at time t, consumes $C_{1,t}$ and accumulates $A_{2,t}$ assets for the second period of life. The agent can invest in equities that pays a risky rate of return equal to e_t during period t and the agent can invest into bonds that pays a constant riskless rate of return equal to r. The individual also faces a Social Security payroll tax equal to τ, the proceeds of which are invested into three types of assets: bonds, equity and a pay-as-you-go Social Security asset that pays a rate of return equal to g known at time t. See the Appendix for the corresponding consumer's maximization problem and the specification of technology.

Under the DC risk allocation in this section, an agent receives whatever the equity-based trust fund pays out, whether large or small. The individual in period 2 therefore has assets equal to[7]

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\begin{aligned}
(1) \qquad A_{2,t} &= \left[(1-\tau)Y_1 - C_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\
&\quad +\tau Y_1\left[(1+r)\beta_r + (1+e_t)\beta_e + (1+g)(1-\beta_r-\beta_e)\right] \\
&= \left[Y_1 - C_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\
&\quad +\tau Y_1\left[(1+r)(\beta_r-\alpha) + (1+e_t)(\beta_e+\alpha-1) + (1+g)(1-\beta_r-\beta_e)\right]
\end{aligned}
$$

where α ($0 \leq \alpha \leq 1$) is the share of private assets invested into bonds and ($1-\alpha$) is the share of assets invested in equities. β_r is the share of the payroll tax invested into bonds, β_e is the share of the payroll tax invested in equities (where $\beta_r + \beta_e \leq 1$) and the remainder, ($1-\beta_r-\beta_e$), is the share of the payroll tax invested into the pay-as-you-go asset. Under the current system, $\beta_r > 0$ and $\beta_e = 0$. Time subscripts for these portfolio share variables are omitted.

Notice from equation (1) that the individual already effectively controls exactly how his/her payroll tax, τY_1, is divided between equity and debt. Specifically, the agent chooses α in order to obtain his/her optimal values of ($\beta_r-\alpha$) and ($\beta_e+\alpha-1$), the shares of Social Security invested into risk-free debt and equity, respectively. Hence, any change in the investment strategy of Social Security that keeps the pay-as-you-go portion constant—e.g., investing the trust fund in equities ($\Delta\beta_r = -\Delta\beta_e$)—has no impact on the combined (public and private) portfolio allocation of this agent. Intuitively, agents see the trust fund simply as their conduit for investing into bonds or equity and

[7] The budget constraint can be easily enhanced to include inherited assets. Ricardian equivalence would not nullify the fact that the ASY and DB risk allocations imply a hidden tax increase on future generations but Ricardian equivalence would imply that households would realize it (maybe because of this paper) and would choose to offset it.

so they take this into account in their private investment decisions. This leads to the following proposition, which is proven formally in the Appendix:

PROPOSITION 1 *Investing the Social Security trust fund in equities under the DC risk allocation constitutes a "shell game" that does not alter the share of aggregate assets devoted to equity or bonds, nor has any price effects.*[8]

The obvious concern with the DC risk allocation therefore is that it does nothing of substance, including failing to boost the expected combined (private and public) retirement benefits relative to the current tax rate. Even Social Security retirement benefits themselves are not necessarily maintained at their current levels since all of the risk is devoted to a single generation whose benefits therefore are risky and depend on market performance—identical to a personal saving account. As noted in Section I, this option was explicitly rejected by many proponents of investing the trust fund in equities.

Proposition 1 also counters an incorrect yet widely held belief by those who adhere to the shell game scenario; namely, that a shift in the trust fund investment strategy from bonds to equity will have, possibly detrimental, price effects. For example, the recent 1997 Economic Report of the President (ERP) writes that,

> Another criticism of allowing the trust funds to invest in equities is that such investments would primarily represent a reallocation of assets between those held in the trust funds and those held—either directly or indirectly—by households. It could improve the financial position of the trust funds, because of equities' historically higher average returns, but for a given level of saving it would not increase the returns for the Nation as a whole. Investing a portion of the trust funds in equities would raise the price and lower the return on equities, and lower the price and raise

[8] Technically, assumptions A3.3 - A4 are not needed for the shell-game scenario.

> the return on Treasury securities. Higher Treasury yields would raise Federal interest costs and, all else equal, the non-Social Security portion of the deficit.[9]

As Proposition 1 proves, however, investing the trust fund in equities will not have any price effects. The reason is that households will want to continue to hold the exact same combined (public and private) level of equities and bonds after the trust fund is invested in equities. Households therefore are willing to hold these assets at the same prices as before. Investing the trust fund in equities is a pure swap between privately and publicly held assets with no need for prices to adjust.

But if investing the trust fund in equities *did* have price effects, then the case for investing the trust fund in equities presumably takes on *more* weight, not less. The reason is that, under the DC risk allocation, any movement in prices may indicate that households do not currently view the trust fund as their conduit for holding bonds. It follows that the Social Security Administration (SSA) could possibly improve the welfare of these "information constrained" people by seeking a better balance between risk and return for these people. This may be the scenario that some members of the recent Advisory Council had in mind when they wrote:

> Currently it can be argued that the government is not performing its role as fund manager for Social Security as well as it might—not because of any failure on the part of those managing the system but because Social Security by law is allowed to invest only in the most conservative of investments: long-term, low-yield government bonds. Trustees of private pension systems and managers of state pension systems, who have the authority to invest much more broadly, would surely be castigated if they pursued such an ultra-conservative investment policy, and it can be argued that Social Security should have the same freedom to invest part of its funds in the broad equities market representing practically the entire American economy.[10]

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[9] The 1997 *Economic Report of the President*, p. 113.

[10] *Findings, Recommendations, and Statements*, p. 83.

To be fair, the *Economic Report of the President* may not have had the *pure* shell game scenario in mind in their analysis. Moreover, they can be commended for stating that—as a first-order effect—investing the trust fund in equities amounts to little more than a reshuffling of assets. But the emphasis on prices—regardless of the risk allocation in mind—is a little wide of the mark in the absence of non-pecuniary externalities. *The real issue from a societal perspective is not the price effects associated with investing the trust fund into equities but how the associated risk is allocated.*

2. The Asymmetric (ASY) Allocation: An Intergenerational Transfer of a Put Option from the Young to the Old

Under the ASY risk allocation, any shortfall in the equity portion of the trust fund below expectation accrues to future taxpayers in the form of higher taxes while any return at, or in excess of, expectation is passed on to current taxpayers in the form of higher benefits.

The Derivation of the Put Option Transfer

Let $\bar{e}_t$ denote the time-*t* rate of return to equity expected immediately prior to time *t*: $\bar{e}_t = E(e_t|t)$. By assumption, the payroll tax is raised on first-period agents in order to fully absorb any shortfall below expectation in the previous period's value of the equity portion of the trust fund. This *additional* payroll tax, denoted as τ^a, equals, at time *t*,

(2)
$$\tau_t^a = \frac{\tau Y_1 \beta_e \cdot \max(0, \bar{e}_{t-1} - e_{t-1})}{Y_1(1+g)} = \tau \beta_e (1+g)^{-1} \cdot \max(0, \bar{e}_{t-1} - e_{t-1})$$

where the numerator equals the shortfall in the value of the equity portion of the trust fund while the denominator is the growth-adjusted tax base of the current first-period agents.

The individual born a time t has assets at age 2 equal to

(3)
$$\begin{aligned} A_{2,t} &= \left[(1-\tau-\tau_1^a)Y_t - C_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\ &\quad +\tau Y_1\left[(1+r)\beta_r + \left[1+e_t+\max(0,\bar{e}_t-e_t)\right]\beta_e + (1+g)(1-\beta_r-\beta_e)\right] \\ &= \left[Y_1 - C_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\ &\quad +\tau Y_1\left[(1+r)(\beta_r-\alpha) + (1+e_t)(\beta_e+\alpha-1) + (1+g)(1-\beta_r-\beta_e)\right] \\ &\quad -\tau_t^a Y_1\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\ &\quad +\tau_{t+1}^a Y_1(1+g) \end{aligned}$$

The second equality in equation (3) demonstrates that the risk allocation considered in this section is *not* a shell game. In general, the third and fourth terms of the second equality will not add up to zero and so the shell game scenario is no longer true.

Equation (3), however, is only the *realized* budget constraint and so it therefore cannot be used to determine the value of this risk allocation. More generally, first-period agents face an *uncertain* value of e and so they face an *expected value* budget constraint at the beginning of their life. Equation (3) implies an expected value budget constraint but not vice versa.

Whereas investing the trust fund in equities along with the DC risk allocation did not change the realized nor the expected budget constraint, the ASY risk allocation considered in this section does. Conceptually, one can think investing the trust fund in equities along with the ASY (or DB) risk allocation as a two-step process. In the first step, the government invests the trust fund in equities and announces the DC risk allocation. In the second step, the government announces a switch from the DC to the ASY (or DB) risk allocation. As shown in Section 1, the first step has no impact on any variables in the economy. *All of the change in economic variables comes solely from the change in the risk allocation.* The change in the *realized* budget constraint therefore comes from subtracting equation (1) from (3). The corresponding change in the *expected* value budget constraint measured immediately prior to the first period of life for a person born at time t, therefore equals

(4)

$$\begin{aligned}
\Delta_p &= E\left(\frac{\lambda U'(C_{2,t})}{U'(C_{1,t})}\cdot(1+g)\tau^a_{t+1}Y_1 \mid t\right) - (1+r)\cdot E\left(\frac{\lambda U'(C_{2,t-1})}{U'(C_{1,t-1})}\cdot\tau^a_t Y_1 \mid t-1\right) \\
&= \beta_e\tau Y_1\cdot E\left(\frac{\lambda U'(C_{2,t})}{U'(C_{1,t})}\cdot\max(0,\bar{e}_t-e_t)\mid t\right) - \frac{1+r}{1+g}\cdot\beta_e\tau Y_1\cdot E\left(\frac{\lambda U'(C_{2,t-1})}{U'(C_{1,t-1})}\cdot\max(0,\bar{e}_{t-1}-e_{t-1})\mid t-1\right) \\
&= \beta_e\tau Y_1\cdot P^P(e_{t-1},1) - \frac{1+r}{1+g}\cdot\beta_e\tau Y_1\cdot P^P(e_{t-2},1) \\
&= \left(1-\frac{1+r}{1+g}\right)\cdot\beta_e\tau Y_1\cdot P^P(e,1) \quad \textit{within stochastic steady state}
\end{aligned}$$

where $U'(\cdot)$ is the marginal utility of any valid utility function, $\lambda \equiv 1/(1+\rho)$ where ρ is the rate of time preference, and $P^P(e_{t-1},1)$ is the price of a one-period put option with a strike price of \$1$(1+\bar{e})$ at time t on a dollar's worth of equity that paid a rate of return equal to e_{t-1} in the previous period.[11] The above formula is simplified by assuming $g_t = g_{t-1}$ although it can easily be rewritten with $g_t \neq g_{t-1}$ with no change to the results herein. Reducing the equations to their steady-state forms is also done purely for illustrative simplification.

Turning to the first equality of equation (4), the first term in this equality equals the value of the promise that is received in the first period of life by a generation-t agent corresponding to generation (t+1)'s obligation to make up for any shortfall in the growth of the trust fund below its expected value at time t via an increase in the payroll tax. The second term of the first equality equals the value of the promise that must be paid by generation t to make up for any shortfall in the growth of the trust fund for the t-1 generation. Assumption A3.3—the existence of an

[11] See Sumru Altuğ and Pamela Labadie (1994) for an excellent discussion on pricing derivative instruments.

intergenerational futures market—is being employed in the second term of this equality and is the reason why the value of this intergenerational insurance contract is of equal value to current and future workers at the margin.

The second equality of equation (4) comes from substituting equation (2) into the first equality in equation (4).

The third equality of equation (4) recognizes the fact that the $E(\bullet)$ expression is equal to the price of a one-period put option for a dollar's worth of stock with a strike price equal to the expected value of this stock, $\$1(1+\bar{e})$. This equality has a very intuitive interpretation. Generation-t individuals receive free put options in the first period of their lives for every dollar they invest in equities via the trust fund. This option underwrites the equity portion of the trust fund in full, guaranteeing that these individuals will receive a rate of return of at least $\bar{e}_t$. This guarantee is represented by the first term on the RHS of this equality. The second term of this equality reflects the fact that this same person was pre-committed by the previous generation to give away a put option that underwrites a poor performance of the equity portion of the trust fund below its expected value incurred during the previous period. The total amount needed to be underwritten—equal in value to the total size of the portion of the trust fund invested in equities by the previous generation—is smaller by the fraction $(1+g)$ of the amount that the next generation will have to underwrite due to the growth in the tax base. On the other hand, since this stock option is being paid for immediately prior to being potentially exercised—instead of being purchased in the previous period—the effective cost of this option is increased by $(1+r)$ to reflect the implicit borrowing cost.

The last equality of equation (4) reflects the fact that in stochastic steady state, the higher moments of the equity price is constant and so the value of the put option is also constant. This equality demonstrates that the asymmetric ASY risk allocation considered in this section is equivalent to increasing the pay-as-you-go portion of the Social Security program by the value of a stock put option that fully insures the equity portion of the trust fund against a performance below its expected value. Just like increasing the size of the pay-as-you-go portion of Social Security, the ASY risk allocation gives the initial generation an instant windfall at the cost of decreasing the expected value of the lifetime budget constraints for subsequent generations when $r > g$.[12,13] This risk allocation scheme would also lead to a reduction in national saving relative to the baseline of maintaining the current payroll tax rate. The key results of this section are summarized in the following proposition:

PROPOSITION 2 *Investing the trust fund in equities under the ASY risk allocation scheme is equivalent to increasing the pay-as-you-go portion of the Social Security program by the value of a stock put option that fully insures the equity portion of the trust fund against a performance below its expected value. Relative to a baseline policy of maintaining the current payroll tax rate, the ASY risk allocation represents an instant windfall to current workers, contracts the expected value budget constraint for future workers (for $r > g$) and reduces national saving.*

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3. The Defined-Benefit (DB) Risk Allocation: An Intergenerational Transfer of a Futures Contract from the Young to the Old

This section investigates the DB risk allocation, which entails investing some of the trust fund in equities with a rate of return guaranteed by future workers to exactly equal the expected

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[12] If, on the other hand, $r < g$ then it is inefficient to have a trust fund.

[13] Note that the internal rate of return to Social Security, g, is compared against the risk-free rate r, instead of against $\bar{e}$ or some blended rate. Intuitively, while the path of the stock price itself is non-differentiable, the value of the stock *option* itself is certain.

return. Specifically, any shortfall in the equity fund's expected value accrues to future taxpayers in the form of higher taxes (just like the previous section) while any return in the equity's fund in *excess* of its expected value accrues to future taxpayers in the form of lower taxes (versus higher benefits as in the previous section). The DB risk allocation therefore generates a guaranteed additional amount of revenue to the Social Security Administration equal to $\tau Y_1 \beta_e \cdot [E(e) - r]$ (where $\Delta\beta_e \leq -\Delta\beta_r$)—i.e., the amount of Social Security revenue invested in equities times the wedge between the new guaranteed rate of return and the current rate of return. This increase in revenue can then be used to help maintain current law benefits for current workers independent of short-term equity returns and without an explicit pre-determined increase in the payroll tax.

Deriving the Futures Contract Transfer

The *additional* payroll tax τ^a under the DB risk allocation is redefined to equal, at time t,

(5)
$$\begin{aligned} \tau_t^a &= \tau \beta_e (1+g)^{-1} \cdot (\bar{e}_{t-1} - e_{t-1}) \\ &= \tau \beta_e (1+g)^{-1} \cdot \left[\max(0, \bar{e}_{t-1} - e_{t-1}) - \max(0, e_{t-1} - \bar{e}_{t-1}) \right] \end{aligned}$$

Notice that equation (5) is identical to equation (2) except with the addition of the second term in the square brackets reflecting the fact that, in the DB plan, any equity return above expectation leads to a reduction in the payroll tax rather than an increase in benefits (as in ASY).

The DB risk allocation is identical to the ASY risk allocation except with the addition of an intergenerational transfer of a call option from current workers to future workers, reflecting the fact that any return to the equity portion of the trust fund in excess of expectation reduces the payroll tax rate. To prove this, substitute (5) into the first equality of equation (4) and reduce to get the following expression (with Δ_p replaced with Δ_f):

(6)

$$\Delta_f = \left[\beta_e \tau Y_1 \cdot P^P(e_{t-1},1) - \beta_e \tau Y_1 \cdot P^C(e_{t-1},1)\right] - \frac{1+r}{1+g} \cdot \left[\beta_e \tau Y_1 \cdot P^P(e_{t-2},1) - \beta_e \tau Y_1 \cdot P^C(e_{t-2},1)\right]$$

$$= \beta_e \tau Y_1 \cdot \left[\frac{1+\overline{e}}{1+r} - 1\right] - \frac{1+r}{1+g} \cdot \beta_e \tau Y_1 \cdot \left[\frac{1+\overline{e}}{1+r} - 1\right] \quad \textit{within stochastic steady state}$$

$$= \left[1 - \frac{1+r}{1+g}\right] \cdot \beta_e \tau Y_1 \cdot \left[\frac{1+\overline{e}}{1+r} - 1\right]$$

The first term on the RHS in the first equality in equation (6) reflects a put and call option received and written, respectively, by a generation-*t* agent constituting a futures contract with generation-(*t*+1) agents to insure that the generation-*t* agent receives a rate of return on his trust fund investment equal to $\overline{e}_t$ regardless of how the fund actually performs. The second term on the RHS in the first equality in equation (6) reflects the futures contract that must be paid by the generation-*t* agent that insures that the generation-(*t*-1) also receives a rate of return on their trust fund investment equal to $\overline{e}_{t-1}$. As before, assumption A3.3 can be seen in the second term of the first equality and is the reason that the value of this intergenerational contract is of equal value to current and future workers at the margin.

The second equality in equation (6) can be derived directly from the agent's first-order conditions or, in terms of option pricing, comes from substituting into the first equality in equation (6) the Hans Stoll (1969) put-call parity relationship for a one-period contract with strike price equal to the expected price and a current price equal to \$1:

(7) $$P^P(e_t,1) - P^C(e_t,1) = (1+\overline{e}_{t+1})/(1+r) - 1 \quad .$$

The third equality in equation (6) simplifies the second equality.

The expression $(1+\bar{e})/(1+r)-1$ in equation (6) is the shadow risk premium associated with the futures contract. Specifically, the expression $(1+\bar{e})/(1+r)-1$ is the premium that would be necessary to give to future generations in order to entice them to *voluntarily* accept the gamble associated with the DB risk allocation. The premium is zero if every taxpayer of the next generation is risk neutral ($r = \bar{e}$). If some future taxpayers are risk averse ($r < \bar{e}$) then the risk premium is positive even for $r = 0$ because the futures contract price is set at a value that is higher than the current price plus risk-free interest; or, equivalently, because the put option is, in present value terms, "in the money" (has a present-value strike price higher than the current price) while the conjoining call option is "out of the money." Because both option contracts are not "at the money," accepting them represents a gamble and the expression $(1+\bar{e})/(1+r)-1$ is simply equal to the risk premium associated with taking this type of gamble that replaces a certain payroll tax with an uncertain value.

Notice that the arrangement of risk considered in this section is equivalent to increasing the pay-as-you-go portion of the Social Security system by the value of the futures contract that insures that the equity portion of the trust fund performs exactly as expected. Similar to the ASY risk allocation considered in the previous section, the DB risk allocation reduces the expected value budget constraint when $r < g$, represents an instant windfall to the initial generation of workers, and reduces national saving. The key results of this section are summarized in the following proposition:

PROPOSITION 3 *Investing the trust fund in equities under the current-law DB risk allocation is equivalent to increasing the pay-as-you-go portion of Social Security by the value of a futures contract for the equity portion of the trust fund with a contract price equal to the expected value of the fund. The value of this contract is positive unless all future taxpayers are risk neutral. Relative to maintaining the current payroll tax, the DB risk allocation gives an instant windfall to current* *workers, shrinks the expected value budget constraint for future workers (when $r > g$) and reduces national saving.*

The DB risk allocation considered in this section leads to an actuarial increase in the pay-as-you-go portion of Social Security that is smaller than the increase corresponding to the ASY risk allocation. The reason is that, while future taxpayers are committed to finance any shortfall in the trust fund's value below its expected value in both cases, future taxpayers, under the DB plan, receive a benefit, in the form of lower taxes (the call option), if the trust fund performs better than expected. The following proposition is proven in the Appendix:

PROPOSITION 4 *The DB risk allocation leads to a smaller actuarial increase in the pay-as-you-go portion of the Social Security system relative to the ASY risk allocation.*

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4. Raising Taxes Versus Passing the Risk

The analysis of the previous two sections demonstrates that investing a portion of the trust fund in equities in an attempt to maintain benefits at a level that is higher than that which can be supported with the current tax rate is mathematically equivalent to increasing the tax rate anyway.

Two related questions therefore arise. First, could future taxpayers actually be better off if the trust fund simply continued to invest into government debt and the government instead maintained the current level of benefits by increasing tax rates *tomorrow* (i.e., on only future taxpayers)? Second, what if, instead, the government increased tax rates *today* in order to tax not only future taxpayers but present taxpayers as well?

Raising Taxes Tomorrow

Consider a policy in which the payroll tax rate on future taxpayers is explicitly increased in order to generate an increase in the level of revenue equal to the increase expected to be generated from investing the trust fund in equities. The resulting change in the present value of the expected value budget constraint resulting from this explicit tax increase is proven in the Appendix to equal

(8)

$$\Delta_t = -\left[\frac{1+r}{1+g}\right]\left[\frac{1+\bar{e}}{1+r}-1\right]\cdot\beta_r\tau Y_1 + \frac{(1+g)}{(1+r)}\cdot\left\{\left[\frac{1+r}{1+g}\right]\left[\frac{1+\bar{e}}{1+r}-1\right]\cdot\beta_r\tau Y_1\right\} \quad \textit{within steady state}$$

$$= \left[1 - \frac{1+r}{1+g}\right]\cdot\left[\frac{1+\bar{e}}{1+r} - 1\right]\cdot\beta_r\tau Y_1$$

The first term in equation (8) reflects the increase in the pay-as-you-go taxes that must be paid in the first year of life in order to achieve an increase in the level of Social Security benefits obtained if the trust fund earned a rate of return equal to $\bar{e}$ instead of r. The second term reflects the discounted rate of return to this pay-as-you-go tax increase that is received by this agent in the second period.

Raising Taxes Tomorrow versus The DB and ASY Risk Allocations

The change in the expected value budget constraint resulting from increasing taxes tomorrow, Δ_t, is exactly equal to Δ_f, the change resulting from investing the trust fund in equities under the DB plan (with β_r in equation (8) now set equal to β_e in equation (6)). Hence, both plans place an equal burden on future taxpayers! This equality extends trivially to general equilibrium since both policies have an identical impact on the budget constraints on each generation.

Notice that this rather remarkable equivalence relationship is completely independent of the functional form assumed for the utility function—including a linear utility function corresponding to risk neutrality. In the case of risk neutrality ($r = \bar{e}$), the value of futures contract is zero and so via equation (6), $\Delta_f = 0$. But since the trust fund is already obtaining $r = \bar{e}$, no additional pay-as-you-go tax revenue is needed to achieve a level of revenue equal in value to investing the trust fund in equities paying an expected rate of return equal to $\bar{e}$. Hence, via equation (8), $\Delta_t = 0$. As the values of r and $\bar{e}$ begin to diverge—i.e., as agents become more risk averse—the value of the risk premium associated with investing the trust fund in equities moves in lockstep with the value of the increase in the equal-revenue pay-as-you-go tax rate.

By Proposition 4, the actuarial tax liability associated with the ASY plan is even *more* expensive to future generations than the explicit payroll tax increase just considered.

The Value of the DB and ASY Risk Allocations

I now price the value of the DB risk allocation followed with ASY plan. Assume that generations are separated by 25 years and assume a per-annum value of $\bar{e}$ equal to 0.07 (the long-run historical annual geometric average equity return to the S&P500 adjusted to include dividend income). These choices imply that each dollar of the trust fund invested in equities during an agent's working life is expected to be worth \$5.43 (=\$$[1.07]^{25}$) by the next period. Also assume a value of r equal to 0.02 (the post-war geometric average real rate of return obtained by the trust fund).

By equation (6), each dollar of the trust invested into the equity under the DB allocation therefore places an actuarial tax liability onto future workers equal to \$2.31 (=\$$[(1+\bar{e})^{25}/(1+r)^{25} - 1]$) or \$3.79 (=\$2.31$\cdot[1+r]^{25}$) in future dollars. But under its current investment policy, each dollar in the trust fund would have been worth \$1.64 (=\$$[1.02]^{25}$) by the next period which added to \$3.79 gives \$5.43. Hence, most of the \$5.43 *absolute* increase in the value of the trust fund comes the \$3.79 actuarial tax on future workers. Of course, all of the \$3.79 increase coming solely from switching from bonds to equity comes at the expense of future workers. As the previous analysis shows, it would have been no different in actuarial terms if the government simply kept the trust fund invested into bonds and instead raised \$3.79 in new taxes from future workers for each dollar in the trust fund that the government would have invested in equities.

To price the value of the ASY plan, an additional assumption about the stock price evolution is required in order to decompose the value of the risk premium stated in equation (7) into its separate stock call and put option components. If we assume that the stock price evolution is geometric Brownian motion (see the additional assumptions stated in the Appendix), we can use the Black-Scholes (1973) option pricing formula to compute the value of the call option. The innovation of the Black-Scholes formula is that it is derived from an arbitrage relationship that incorporates current market values reflecting all underlying parameters explaining the premium people place on risk. Equation (7) can then be used to impute the value of the put option representing the actuarial cost of the ASY plan. As a measure of volatility, I use the second moments of the S&P500 time series (please see the Appendix). Using the parameter values for $\bar{e}$, r and the time period noted above, I computed the value of call option on a \$1 stock with a strike price of \$5.43 and a maturity date of 25 years to equal only \$0.14. The value of the put option then equals \$2.45 in current dollars.

Putting these pieces together we see that the actuarial tax placed on future workers resulting from the DB plan is only slightly less than the tax associated with the ASY plan (\$2.31 versus \$2.45 in current dollars). In other words, the reward to future workers associated with the DB plan in the form of promising future workers returns in excess of the expected return (the call option) is almost worthless relative to the costs imposed on future workers in the form of a put option. Intuitively, and in the language of option pricing, since the strike price equals the *expected value* of the trust fund, the implied put option component of the DB plan is significantly "in the money" while the implied call option component is significantly "out of the money."

What these calculations imply is that, if future generations are obligated to make up for a short fall in the value of the trust fund below its expected value, policymakers today need not worry whether future policymakers switch the risk allocation from DB to ASY. Moreover, if policymakers announce DB today, they need not worry if politically-astute economic agents—aware of the graying of the median voter—will choose to optimize as though something closer to ASY has been actually been enacted. The key issue for intergenerational equity is whether future generations will be obligated to finance a shortfall rather than how returns in excess of expectation will be distributed.

Raising Taxes Today

Table 2 demonstrates that immediately increasing the payroll tax rate, instead of waiting until the cost of Social Security exceeds its income, will result in an increase in the payroll tax rate for someone born twenty five years from today that is one-half the tax increase that will be needed once outlays begin to exceed costs.[14] Specifically, under, for example, the intermediate demographic and economic assumptions assumed in the 1997 OASDI Trustees Report, a 75-year balance can be achieved with an immediate increase in the payroll tax by 2.23 percentage points. However, adopting a rule of increasing payroll tax rates only when costs exceed revenue will require an increase in the payroll tax rate of 4.33 percentage points in twenty five years and 5.38 percentage points in 50 years. It is clear therefore that future taxpayers would be much better off in actuarial terms if payroll taxes are raised immediately versus waiting until costs exceed income.

In light of the equivalency between the DB plan and the explicit tax increase on only future taxpayers, future taxpayers are therefore better off in actuarial terms if tax rates were raised immediately versus investing the trust fund in equities and requiring future taxpayers to bear the downside risk in the form of higher tax payments. Of course, increasing the payroll taxes

[14] Of course, technically, the Social Security Administration does not need higher tax revenue until the trust fund itself runs out (around 2030). But waiting until then will require an even higher tax rate.

immediately is not the only option. The government could also adjust the growth rate of costs. But what the analysis herein does show is that rejecting increasing the payroll tax rate in favor of investing the trust fund in equities is without merit.

I summarize the results in this section in the following proposition:

PROPOSITION 5

1. *Investing the trust fund in equities under the current-law DB Social Security system places a burden on future generations equal in actuarial value to raising the same extra expected revenue by explicitly taxing only future generations.*
2. *Future generations are worse off under the ASY risk allocation relative to raising the same expected revenue by taxing only future generations.*
3. *Future generations would be better off if the government raised the same expected revenue by immediately increasing taxes rather than investing the trust fund in equities and allocating the risk either according to the current-law DB plan or the ASY plan.*

III. Relaxing the Assumptions[15]

It is well know that many households hold little or no financial wealth. This stylized fact does not matter under assumptions A1- A4 made in Section II, but it becomes more important upon relaxing those assumptions. This section discusses how the conclusions of the previous section change upon relaxing each assumption. As shown herein, relaxing the assumptions does not necessarily improve the case for investing the trust fund into equities. Indeed, the analysis of Section

[15] Comments by Peter Diamond resulted in a significant improvement in the material presented in this section.

II might be optimistic.

1. The DC Plan

While the DC risk allocation was neutral under the earlier assumptions, there are several reasons to suspect that it might lead to an improvement in efficiency. First, "information constrained" people might not recognize that the trust fund is currently a conduit for their own saving. Second, some people might pick stocks poorly or stay out of the market altogether due to limited knowledge. Third, trading costs might keep people out of the market (although the low trading costs observable in the market makes this scenario less likely[16]). Fourth, the current trust fund might be forcing borrowing-constrained people to hold too much risk-free assets.[17]

There are other reasons, however, why the DC plan might hurt informed agents. These reasons include the presence of short-sale restrictions and capital gains taxation which reduces the ability to offset changes in the trust fund investment strategy. Incorporating intragenerational heterogeneity and the redistributional mission of Social Security might weaken the case for equity investment even more, especially if the government offsets losses by simply shifting downward the progressive replacement rate function by a proportional factor. The DC plan therefore involves tradeoffs. Another concern about the DC risk allocation is more political in nature: is it credible to believe that the government is really willing to reduce Social Security benefits on retired people if their equity investment does not perform as expected? Or, would a DC plan evolve to a ASY or DB

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[16] In terms of transaction costs, most passive-indexed non-institutional equity funds now have administrative costs of only around 20 basis points. Equity funds that focus on institutional investors charge only about one-half of this—about the same amount charged by the Thrift Savings Plan for federal government employees and many 401(k) accounts. See Olivia Mitchell (1996). Not only can many workers invest a portion of their payroll into a 401(k) account, one can also invest into a no-load mutual fund with an initial investment of $250 and $50 a month.

[17] On the other hand, a strict pay-as-you-go design may be more efficient than having a trust fund in the presence of non-tradeable human capital (Merton [1983]).

plan in practice? This is an especially important question in light of the graying of the median voter.

2. The ASY and DB Plans

The four possible sources of efficiency gains noted above for the DC plans also apply to some degree to the ASY and DB plans. However, these potential gains are fully obtainable within the DC plan and a movement from the DC risk allocation to either the ASY or DB risk allocation still places a large actuarial tax burden on future generations regardless of whether future generations are aware of it. Moreover, the previous analysis might have underestimated the burden on future generations associated with the ASY and DB plans by assuming that the trust fund is perpetual. In reality, the trust fund exists to buffer the coming demographic problem. An ambitious switch in the trust fund investment policy toward equity would extend the life of the trust fund by 8 years over baseline—from the year 2029 to the year 2037—*if* equity performed as expected.[18] Yet no agent born after 1974 could qualify for benefits prior to 2036. Under the ASY and DB risk allocations, these agents would get stuck with the actuarial costs but with little of the benefits resulting from risk reduction, except for disability insurance which is collected before retirement. Relaxing assumption A4 therefore weakens the case for investing the trust fund in equities.

Although the ASY and DB plans impose an actuarial cost on future workers equal to the amounts stated in Section II, the *total* cost of the additional risk shifting might be less upon relaxing the complete market assumption A3.3 *if* future workers are inadequately exposed to contemporaneous shocks. Indeed, probably the most compelling argument for investing the trust fund in equities under these type of plans is the possibility given by the Technical Panel quoted in

[18] See Thomas Jones (1995). As he shows, however, it is possible that a combination of trust fund investment changes *and* tax increases or benefit cuts might be able to keep the trust fund active for the foreseeable future. This route has, in fact, been taken by those members of the recent Advisory Council advocating trust fund investment.

Section I: "Holding equities in the fund could give future generations exposure to current stock returns, thus improving risk sharing across generations." It should be noted, however, that the Technical Panel simply stated this as a possibility—rather than as definitive conclusion—and they proposed that additional study be done. (Whether investing the trust fund in equities is the most efficient way of completing this missing market is outside the scope of this paper.)

It is also true, however, that the previous analysis might have *understated* the cost of the ASY and DB plans and, in particular, future workers might be more willing to accept the explicit tax increase on only them versus the DB plan (and therefore the ASY plan) that exposes them to more risk. As evidence, note that the pay-as-you-go portions of the defined-benefit Medicare and Social Security program alone create an enormous amount of tax rate uncertainty for future workers. For example, the difference between the low and high cost projections in the 1997 OASDI Trustees Report (see Table 3) demonstrates a tremendous amount of uncertainty in future tax rates driven by demographic, economic and longevity factors alone.[19] Debt policy creates another source of intergenerational risk shifting as does taxing capital income. Another issue is the timing of wage contracts: shifting some of the effects of negative shocks to future workers becomes less appealing the longer it takes for negative shocks to fully translate into lower wages. As noted earlier, the answer to the question whether the complete market analysis of Section II paints an overly pessimistic or optimistic picture therefore depends on whether current workers face too little or too much exposure to risk relative to future workers—a very ambitious but important project.

[19] It is sometimes argued that low and high cost ranges of the Trustees Report exaggerate the potential variation because the ranges are computed using the optimistic and pessimistic projections, respectively, for *each* underlying forecasting variable. This effect, however, is partially offset by the Trustees' focus on a "reasonable" range for each forecasting variable (see pages 145-68 of the 1997 Trust Fund report). Since values outside of these ranges are quite possible and since some parameters (e.g., average earnings) can alone be very important, one should not interpret the high and low cost estimates as envelopes. Zeldes (1995, p. 403) makes a similar point.

In any case, neither the DB or ASY plans can result in efficiency gains even if future workers are underexposed to current stock price risk. The reason is that neither plan rewards future workers for taking additional risk in the form of a positive expected payoff. This result was proven formally in a simple incomplete market model in an earlier version of this paper and Bohn (1997*b*) has reached the same conclusion using a more general incomplete market model. A necessary—but not a sufficient—condition for efficiency gains therefore is that future generations are asked to guarantee a rate of return less than the expected rate-of-return to equity, thereby giving future workers a positive expected return. But this raises the thorny issue of whether future policymakers will really be willing to distribute returns in excess of some previously specified level to future taxpayers in the face of a graying median voter or whether such a complex plan would reduce to the DB or ASY plan in practice.

IV. Conclusion

This paper demonstrated that, in a complete market setting, investing the Social Security trust fund in equities under a defined-contribution (DC) risk allocation is a pure shell game. Moreover, attempting to maintain the current level of benefits via investing a portion of the trust fund under either the current-law defined-benefit (DB) or the asymmetric (ASY) risk allocation represents a very large actuarial tax on future generations. Indeed, future generations are equally worse off under the DB plan versus an explicit increase in the payroll tax on only future workers that raises the same expected amount of revenue—a result that holds at any level of risk aversion. Future workers are slightly worse off under the ASY plan than the DB plan. Both the DB and ASY plans represent an instant windfall to current workers and reduce national saving relative to a baseline policy of maintaining the current payroll tax rate.

Under imperfect and incomplete markets, there are reasons why investing the trust fund in equities could improve economic efficiency but there are also reasons why it could reduce efficiency. Tradeoffs exist. The question whether the complete market analysis paints an overly pessimistic or an optimistic picture is therefore unresolved. One key related issue left for future research is whether current workers face too much uncertainty relative to future workers under baseline policy. Answering this question is an ambitious project but one that might be worth the effort given its importance.

Section III also raised some political-economy questions. Since any potential efficiency gains are limited to the DC plan or hybrid versions of the DB or ASY cases, it is relevant to inquire whether the government can credibly commit to such complex fiscal policies—especially in light of a graying median voter—or whether or any attempt to do so will simply evolve to the standard DB or ASY policies which unambiguously are not efficiency enhancing. These important political-economy issues are left for future work.

Finally, the option pricing technique can also be used to analyze additional uncertainties associated with various fiscal policies including the numerous risks associated with the Medicare and Social Security defined-benefit programs. This could be an asset to the recent generational accounting literature which tends to focus only on medium demographic and economic projections and therefore tends to underestimate the true actuarial burden placed on future generations.

V. Appendices

Appendix 1: Formal Proof of Proposition 1

Denote the optimal combined (public and private) portfolio share allocations between equity and bonds as $(\beta_r\text{-}\alpha)^*$ and $(\beta_e+\alpha\text{-}1)^*$, respectively. Now consider a change in strategy of the trust fund investment: $\Delta\beta_r = -\Delta\beta_e$. The conditional demand function for α is,

(A1.1) $$\alpha(\cdot) = \beta_r - (\beta_r\text{-}\alpha)^* \quad .$$

I.e., selecting the value of $\alpha = \alpha(\beta_r)$ will generate $(\beta_r\text{-}\alpha) = (\beta_r\text{-}\alpha)^*$ and $(\beta_e+\alpha\text{-}1) = (\beta_e+\alpha\text{-}1)^*$. To prove this result, note that the equality $(\beta_r\text{-}\alpha) = (\beta_r\text{-}\alpha)^*$ holds by substituting (A1.1) into $(\beta_r\text{-}\alpha)$. To prove that the equality $(\beta_e+\alpha\text{-}1) = (\beta_e+\alpha\text{-}1)^*$ holds, substitute $\alpha(\cdot)$ into $(\beta_e+\alpha\text{-}1)$ to get $[- (\beta_r\text{-}\alpha)^* - (1\text{-}\beta_r\text{-}\beta_e)]$. Now the assumption $\Delta\beta_r = -\Delta\beta_e$ implies that the portfolio share devoted to the pay-as-you-go asset has not changed:

(A1.2) $$(1\text{-}\beta_r - \beta_e) = (1\text{-}\beta_r - \beta_e)^*.$$

Substituting (A1.2) into $[- (\beta_r\text{-}\alpha)^* - (1\text{-}\beta_r\text{-}\beta_e)]$ gives $(\beta_e+\alpha\text{-}1) = (\beta_e+\alpha\text{-}1)^*$. These equalities imply that an agent can always obtain his/her original portfolio allocation by applying rule (A1.1). Now to show that the agent will maintain his/her combined (public and private) division of assets between equity and bonds, suppose that is not true. Then a change in β_r (equal to negative the change in β_e) would lead to a new value of share of the payroll taxes invested in equity; call it $(\beta_r\text{-}\alpha)'$. However, $(\beta_r\text{-}\alpha)'$ was already available to the agent prior to the change and the agent revealed his/her preference for $(\beta_r\text{-}\alpha)^*$ over $(\beta_r\text{-}\alpha)'$ (i.e., $(\beta_r\text{-}\alpha)^* \succ (\beta_r\text{-}\alpha)'$). Hence, the agent cannot prefer $(\beta_r\text{-}\alpha)'$ over $(\beta_r\text{-}\alpha)^*$ (i.e., $(\beta_r\text{-}\alpha)' \succ (\beta_r\text{-}\alpha)^* \succ (\beta_r\text{-}\alpha)'$ is a contradiction) after the regime change.■

Appendix 2: The Consumer Maximization Problem and Technology

Consumers live for two periods and a consumer born at time *t* is assumed to have the following preference over a sequence of consumption, $\{C_{j,t}\}_{j=1}^{2}$:

(A2.1) $$E\left\{\sum_{j=1}^{2} \lambda^j U(C_{j,t}) \mid t\right\}$$

where $U(\bullet)$ is the felicity function that is assumed to be continuous, continuously differentiable and strictly increasing along with $U(0) = 0$. Consumers face the following budget constraint (equation (1) in the text):

(A2.2) $$\begin{aligned} A_{2,t} = & \left[(1-\tau)Y_1 - C_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_t)(1-\alpha)\right] \\ & +\tau Y_1\left[(1+r)\beta_r + (1+e_t)\beta_e + (1+g)(1-\beta_r-\beta_e)\right] \end{aligned}$$

The first-order conditions for this problem are:

(A2.3) $$U'(C_{1,t}) = \lambda(1+r_t)E\left[U'(C_{2,t}) \mid t\right]$$

(A2.4) $$U'(C_{1,t}) = \lambda E\left[(1+e_t)U'(C_{2,t}) \mid t\right]$$

Note that for linear utility (risk neutrality), $E[U'(C)]$ is constant and equations (A2.3) and (A2.4) imply that $r = \bar{e}$. It is not possible in general to find a closed-form solution for $\{C_j\}_{j=1}^{2}$, asset variables or α except for very special choices for $U(\bullet)$. Fortunately, however, the analysis presented in the text does not require that any closed-form solution be found; nor does the analysis in the text require an particular form for the utility function.

Net output at time *t* equals $N(K_t, L_t) = F(K_t, L_t) - \delta_t K_t = A_t K_t^{\theta} L_t^{1-\theta} - \delta_t K_t \equiv A_t k_t^{\theta} - \delta_t k_t$ where K_t is the capital stock installed at time *t*, L_t is the labor supply, θ is the capital share and A_t is the economy's efficiency parameter reflecting exogenous labor-augmenting technological change. The homogeneity of $N(\cdot)$ implies that $N(\cdot) = K_t N_t^K + L_t Y_1$ where N^K, the (net) marginal product of capital, is the partial derivative with respect to factor *K* at time *t*. $N_t^K = \phi r + (1-\phi)e_t$ where ϕ is the debt-equity ratio. Hence, (A2.5) $$e_t = (A_t K_t^{\theta} L_t^{1-\theta} - \delta_t K_t - Y_1/k_t - \phi r)/(1-\phi)$$

Shocks can occur to either productivity A or to the depreciation rate δ (what Bohn [1997*a*] refers to as "valuation" risk).

The literature differs on how to set Y_1. Some authors take the labor contract as given at time t (e.g., Blanchard and Fischer [1989, Chapter 6]). This is reasonable if we wish to model equity holders as the full residual claimants to contemporaneous shocks. Other authors allow contemporaneous shocks to alter both capital and labor income at time t (e.g., Bohn [1997*a*]): $Y_1 = N_t^L$ where N^L is the marginal product of labor. This is reasonable if we take each time period as representing several years (as does Bohn). In order to focus primarily on equity risk, the analysis in Section II assumes that the wage growth index, $g_t = Y_{t+1}/Y_t - 1$, is set at the beginning of period t before the value of e_t is known. More general specifications of the wage setting rule would add complexity without altering the key results within a complete market setting. In an *incomplete* market setting, however, greater generality is valuable and the reader is referred to Bohn (1997*a*,*b*).

Appendix 3: Proof of Proposition 4

The result comes from comparing equations (4) and (6) and by noting that $P^P(e,1) > \left[\frac{1+\bar{e}}{1+r} - 1\right]$ by the put-call parity relationship, equation (7).

Appendix 4: Derivation of the Δ_t

Define Δ as follows:

(A4.1) $$\Delta = (1+\bar{e})\cdot\beta_r \tau Y_1 \cdot (1+g)^{-1} - (1+r)\cdot\beta_r \tau Y_1 \cdot (1+g)^{-1}$$

$$= \left[\frac{1+r}{1+g}\right]\left[\frac{1+\bar{e}}{1+r} - 1\right]\cdot\beta_r \tau Y_1$$

Δ is equal to the increase in revenue needed from generation t in order to raise the expected amount of additional revenue that generation t-1 would have received if its trust fund was invested in equities. The expression for Δ_t in equation (8) then equals

(A4.2)
$$\Delta_t = -\Delta + \frac{(1+g)}{(1+r)} \cdot \Delta$$

which reflects the present value of an increase of Δ in the pay-as-you-go Social Security taxes.

Appendix 5: Pricing the Put Option[20]

Suppose the following conditions hold: (1) There are no transactions costs, taxes or restrictions on short sales in the options market. (2) The risk-free rate of interest is constant. (3) The stock pays no dividend. (4) The stock price evolution is geometric Brownian motion. (5) The market is open continuously for trading.

Under these assumptions, the Black-Scholes call option pricing formula is given by

(A5.1)
$$P^{C}(S,t) = S \cdot \Phi\left[\frac{\ln(S/X) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right] - e^{-rt}X \cdot \Phi\left[\frac{\ln(S/X) - \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right]$$

where r = risk-free interest rate, S = current stock price, X = the strike price, σ^2 = variance of changes in the logarithm of the stock price, t = time to maturity, and $\Phi(\bullet)$ is the standard cumulative normal distribution function. The price of the corresponding put option can then be derived using Hans Stoll's (1969) put-call parity relationship:

(A5.2)
$$P^{P}(S,t) = P^{C}(S,t) + X/(1+r)^t - S$$

The value of r was estimated to be about 0.019, the basis for the 0.02 value used in the text.

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[20] This portion of the Appendix closely follows the excellent discussion by Jonathan Ingersoll, Jr. (1989).

To calculate this value, I obtained the effective nominal annual interest rates earned by the assets in the trust fund from 1940 to 1995 from the Office of the Chief Actuary of the Social Security Administration.[21] These values were adjusted to obtain real values using the CPI-U from the Bureau of Labor Statistics along with a correction in measurement changes in the CPI-U series derived from the CBO's "black box." The mean of this series is equal to about 0.019.

The values of the other parameters are set as follows. $S = 1$, $X = (1.07)^{25}$ and $t = 25$. The value of σ was set equal to 0.16 which is equal to the historic standard deviation of the first differences of the natural log of the annual S&P500 index (augmented to include dividend income) from 1949 through 1996.

[21] The data is available from the Social Security Administrations web page and is presented in tabular format at http://www.ssa.gov/OACT/ProgData/effectiveRates.html.

References

1994-1995 Advisory Council on Social Security, The. *Findings, Recommendations, and Statements*.

1997 OASDI Trustees Report, The.

1997 Economic Report of the President, The.

Altuğ, Sumru and Pamela Labadie. *Dynamic Choice and Asset Markets*. Academic Press (San Diego), 1994.

Black, F. and M.J. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 1973, 81(3): 637-54.

Blanchard, Olivier and Stanley Fischer. *Lectures on Macroeconomics.* Cambridge, Mass. and London: MIT Press, 1989, pages xiii, 650.

Bohn, Henning. "Risk Sharing in a Stochastic Overlapping Generations Economy." Mimeo, UC Santa Barbara, June, 1997*a*.

———. "Social Security Reform and Financial Markets." Mimeo, UC Santa Barbara, June, 1997*b*.

Bosworth, Barry. "Fund Accumulation: How Much? How Managed?" in *Social Security: What Role for the Future*. Peter Diamond, David Lindeman and Howard Young, Editors. Distributed by the Brookings Institution for the National Academy of Social Insurance, 1996.

Congressional Budget Office, "Implications of Revising Social Security's Investment Policies." *CBO Papers*, September, 1994.

Fierst, Edith U. "Social Security's Future." Testimony before the Subcommittee on Social Security and Family Policy, Senate Finance Committee, March 25, 1996.

Gale, Douglas. "The Efficient Design of Public Debt." Dornbusch, Rudiger; Draghi, Mario, eds. *Public Debt Management: Theory and History.* Cambridge; New York and Melbourne: Cambridge University Press, 1990, pp. 14-47.

Gramlich, Edward. "Different Approaches for Dealing with Social Security." *Journal of Economic Perspectives*, 1996, 10(3): 55-66.

Greenspan, Alan. "Remarks by Chairman Alan Greenspan." *Abraham Lincoln Award Ceremony of the Union League of Philadelphia*, December 6, 1996.

Hammond, P. Brett and Mark J. Warshawsky. "Investing Social Security Funds in Stocks." Mimeo, TIAA-CREF, 1996. Forthcoming in *Benefits Quarterly*.

Ingersoll, Jonathan. "Option Pricing Theory." The New Palgrave Dictionary of Money & Finance: 199-212.

Jones, Thomas. "Trust Fund Investment Alternatives." In Report of the 1994-1995 Advisory Council on Social Security (Vol. II): 405-414.

Merton, Robert. "On the Role of Social Security as a Means for Efficient Risk Sharing in an Economy Where Human Capital is Not Tradable." Bodie, Zvi; Shoven, John, eds. *Financial Aspects of the United States Pension System*. NBER Project Report Series, Chicago and London: University of Chicago Press, 1983, pp. 325-358.

Mitchell, Olivia S. "Administrative Costs in Public and Private Retirement Systems." NBER Working Paper #5734, 1996.

Stoll, Hans R. "The Relationship between Put and Call Option Prices." *Journal of Finance*; 24(5), Dec. 1969, pages 801-24.

Technical Panel to The 1994-1995 Advisory Council on Social Security. *Technical Panel on Trends and Issues in Retirement Saving*.

Warshawsky, Mark. "Social Security Reform and Financial Markets: Commentary." Mimeo, TIAA-CREF, June, 1997.

White, Lawrence. "Investing the Assets of the Social Security Trust Funds in Equity Securities: An Analysis." *Investment Company Institute Perspective*, 1996, 2(4): 1-20.

Zeldes, Stephen. "Risk in Equity Investment." *Technical Panel to The 1994-1995 Advisory Council on Social Security*, pp. 402-404.

Table 1. Alternative Ways to Allocate First-Period Risk of Trust Fund Investments

	Ex-Post Equity Returns	
	Higher than E(e)	Lower than E(e)
Defined Contribution (DC)		
Current Workers	e	e
Future Workers	0	0
Asymmetric (ASY)		
Current Workers	e	E(e)
Future Workers	0	e - E(e)
Defined Benefit (DB)		
Current Workers	E(e)	E(e)
Future Workers	e - E(e)	e - E(e)

NOTES: e = equity rate in period 1
E(e) = expectation of period-1 equity rate

Table 2. OASDI Income and Cost Rates Over the Next 75 Years
(As a percentage of taxable payroll)

25 Year Period	Income rate	Cost rate	Balance
Intermediate:			
1997 - 2021	12.72	12.76	-0.04
2022 - 2046	13.08	17.41	-4.33
2047 - 2071	13.24	18.62	-5.38
Low Cost:			
1997 - 2021	12.68	11.55	1.13
2022 - 2046	12.95	14.42	-1.47
2047 - 2071	13.01	13.74	-.73
High Cost:			
1997 - 2021	12.77	14.17	-1.41
2022 - 2046	13.26	21.20	-7.95
2047 - 2071	13.61	26.14	-12.52

75 Year Period	Income rate	Cost rate	Balance
Intermediate:			
1997 - 2071	13.37	15.60	-2.23
Low Cost:			
1997 - 2071	13.25	13.03	.21
High Cost:			
1997 - 2071	13.53	19.07	-5.54

Source: 1997 OASDI Trustees Report, Tables I.H1. and I.H2.

Table 3. Combined OASDI and HI Income and Cost Rates Over the Next 75 Years
(As a percentage of taxable payroll)

	25 Year Period		
	Income rate	Cost rate	Balance
Intermediate:			
1997 - 2021	15.78	17.76	-1.98
2022 - 2046	16.34	26.32	-9.98
2047 - 2071	16.59	29.39	-12.79
Low Cost:			
1997 - 2021	15.73	15.68	0.05
2022 - 2046	16.14	19.65	-3.51
2047 - 2071	16.25	19.55	-3.30
High Cost:			
1997 - 2021	15.85	20.53	-4.68
2022 - 2046	16.61	37.18	-20.58
2047 - 2071	17.15	46.85	-29.69

Source: 1997 OASDI Trustees Report, Table III.A3