Can the Standard Growth Model Explain the Post-War Decline in the Saving Rate?

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Abstract

The literature is filled with explanations of why various theories are *not* able to explain the enormous post-war decline in the gross national saving rate in the US and most G-7 nations. But despite these negative results, the previous literature, by focusing only on steady state analysis, has not been able to conclude whether the standard growth model is compatible with the decline. This short paper takes the standard growth model—with and without human capital accumulation—seriously and explores the dynamics of saving along the transition path. The paper investigates whether the process of factor accumulation itself—which, by some estimates, is the source of much of the increase in per-capita income in the US during the past 100 years—could be the cause of the decrease in the saving rate, the idea being that an economy close to steady state may require a relatively lower optimal saving rate.

The paper derives the analytical properties of the endogenous saving rate in the transition path of the standard growth model with CES production, something that has eluded researchers so far. The optimal endogenous saving rate along the transition path might overshoot (undershoot) its limiting value when the factor substitution elasticity between capital and labor is less than (exceeds) unity. The overshooting case is *theoretically* consistent with both the post-war decline in the saving rates and the pre-war increase in the saving rates in most G-7 countries.

Precise calculations of the overshooting case, calibrated to the growth in per-capita income in US during the past century, however strongly reject the notion that the decline in the saving rate can be attributed to transitional dynamics—thus eliminating possibly the last hope for the standard model in explaining the post-war decline in the saving rate. Although it is possible to generate a large decrease in the saving rate induced via factor accumulation, it is not possible to do so in light of the other stylized facts typifying the US growth process during the past century.

In sum, if the standard growth model is to explain the saving rate dynamics during the past century in the US, the model will have to do so with a choice of a production function even more general than CES. More likely however the pure Ricardian nature of the standard model will have to be abandoned in order to explain the post-war decline in the saving rate.

Key words: The Ramsey-Cass-Koopmans-Barro model, Saving, CES production
JEL Codes: E20, O41, O10.

1. Introduction

The gross and net national saving rates decreased by a phenomenal 5-percentage *points* during the previous three decades in the US and in most of other G-7 countries with the exception of Japan and maybe Germany (see Figures 1 - 2 [*a* - *f*]), a conclusion robust to essentially every definition of national saving.[1] To see how important this change in the saving rate is, note that the standard (Ramsey) growth model predicts *conservatively* that an increase in the asymptotic gross saving rate by 5-percentage points (say via a change in the rate of time preference) corresponds to around a 90% increase in the long-run capital stock and a 30% increase in the long-run output per effective labor unit.[2] These numbers eclipse even the upper bound of the estimates found in the literature predicting the potential gains from moving to consumption based taxation.

Any plausible model seeking to explain the post-war decline in the US saving rate must be consistent with at least four stylized facts: [1] the value generated by the model for the interest rate a century ago cannot be too large (King and Rebelo [1993]); [2] the half-life of growth in output per effective labor unit resulting from factor accumulation must be reasonably long—around 40 years or so (Robert Barro and Xavier Sala-i-Martin [1995, Table 11.1]); [3] the saving rate must be non-decreasing—most likely strictly increasing—prior to WWII (the case for the US and most G-7 nations prior to WWII[3]) and [4] the saving rate must be significantly decreasing sometime after WWII (the case for the US and most G-7 nations after WWII [Figures 1 and 2]). Stylized facts [3] and [4] together describe an optimal saving rate that, at least weakly, overshot its long-run value.

The literature seeking to explain the post-war decline in the US saving rate has been dominated by negative results (see the excellent recent literature survey by Martin Browning and Annamaria Lusardi [1996]). There is no consensus regarding the role that post-war <u>demographic shocks</u> have had on the saving rate, either in size or even in sign—and none of the popular studies attributes very much of the decline to demographics.[4] An explanation of the saving rate decrease based on <u>technological shocks</u> would require that future income projections were being revised upward over the past several decades—yet public opinion polls and projections by DRI, the CBO, the CEA and the Blue Chip Consensus Forecast have consistently indicated a *downward* revision in the expected growth rate of income during the past two decades, during a time in which a significant amount of the decline in national saving has occurred. An explanation based on preference shocks would probably be viewed as suspect to most economists if the theory was not accompanied with a clear explanation of why preferences change in such a way to generate the observed non-monotonic path for the saving rate over time. Although changes in precautionary saving can have a potentially important impact on the level of saving in a lifecycle model, it has little importance in the standard Ramsey model where agents can effectively dynamically self-insure against unexpected decreases in income and unexpected increases in required expenditures (Narayana R. Kocherlakota [1996]). The decline in tax rates during the past several decades should have, if anything, lead to an increase in the saving rate.[5] Other changes in the post-war US economy—changes in the distribution of income, big capital gains in housing and asset prices, and financial market innovation—either produce the wrong sign theoretically or have no empirical support (see Martin Browning and Annamaria Lusardi). Still some other researchers have abandoned the standard growth model altogether in favor of the lifecycle model and have argued that increased expansion of old-age support and other changes in the timing of taxes (which are irrelevant in the standard growth model with its Ricardian equivalence property) is the culprit behind the post-war decrease in the national saving rate. The actual evidence supporting this theory is mixed (compare, for example, Laurence Kotlikoff, Jagadeesh Gokhale, John Sabelhaus [1996] with Barry Bosworth [1996]) but it appears to be the most convincing explanation yet. Nonetheless, Browning and Lusardi conclude "that we are still some way from having a convincing explanation of the saving decline."

A limitation of the prior attempts to explain the decrease in the saving rate however is that they tacitly assume that the economy was in steady state prior to the shock or policy change. It is quite possible that factor accumulation itself might play an important role in explaining the saving rate decline. By not considering the transitional dynamics of the standard model explicitly, the previous literature could not reject the ability of the standard growth model in explaining the decline in the post-war saving rate.

The idea that the post-war decline in the US saving rate could be the result of transitional dynamics should not be surprising to most economists trained in standard growth theory. Dale Jorgenson and his colleagues developed a very elaborate econometrics model during the past two decades and they attribute as much as 83% of growth in per-capita income during this century to changes in factor inputs (see, e.g., Dale Jorgenson and Barbara Fraumeni [1989] and references therein). This number implies that the US economy was very far away from its steady state a century ago. Since a realistic transition path in the standard growth takes 40 years or more to converge just 50% of the way to its limiting balanced-path equilibrium—the *initial* 50% at that—it not surprising that it is possible to generate a rich set of long transitional dynamics using the standard growth model (see, e.g., Lawrence Christiano [1989]; Robert King and Sergio Rebelo [1993]; Robert Barro and Xavier Sala-i-Martin [1995]). The question therefore is not whether it is possible to generate a factor-accumulation induced decrease in the saving rate like that observed in the US during the past several decades: this part is easy. Rather, the question is whether it is possible to generate such a decrease that is compatible with stylized facts [1] - [3].

So why haven't researchers in the past considered transitional dynamics as the source of the post-war decline in saving? The reason is that very little is known about the properties of the endogenous saving rate in the transitional economy of the standard growth model. Lawrence Christiano (1989) presents some of the earliest simulation evidence of the transitional behavior of the saving rate for the full Ramsey model. He uses his simulation model to attempt to explain the saving rate dynamics for the post-war Japanese economy. He argues that using a Stone-Geary utility function (where utility is positive only when consumption has exceeded a minimal amount) is a quite reasonable approach for analyzing the saving rate dynamics in the post-war Japanese economy due to the very low consumption among the Japanese after the war. Because the Stone-Geary utility function generates an increase in the growth rate of income along the transition path followed by a decrease—something not true in the post-war US—the saving rate does likewise.

King and Rebelo (1993) present simulation evidence for the Ramsey model but reject the Ramsey model altogether on the basis that it violates stylized facts [1] and [2]. Barro and Sala-i-Martin (1995) are the first to derive actual analytical properties of the endogenous transitional saving rate for the case of Cobb-Douglas production. They also rescue the Ramsey model from the criticisms of King and Rebelo by noting that a large value for the capital share (they use 0.75), meant to include human capital, is compatible with stylized facts [1] and [2]. Barro and Sala-i-Martin however add a third stylized fact to the debate, a fact based on evidence they find in the Summers-Heston panel data set: the saving rate is generally increasing along the transition path. This stylized fact is consistent with stylized facts [3] and [4] above since, by [3], the saving rate is increasing early into the transition process when there is a considerable increase in income while, by [4], the saving rate is decreasing later in the transition process when the increase in income is much smaller. Together, stylized facts [3] and [4] would tend to generate the phenomenon observed in the regression analysis by Barro and Sala-i-Martin.

This short paper derives the analytical properties of the endogenous transitional saving rate for the case of CES technology. This derivation generalizes the analytical results presented recently in Barro and Sala-i-Martin (1995) who utilized Cobb-Douglas production. But the importance of the derivation herein goes well beyond generalizing their work: as Barro and Sala-i-Martin prove, the transitional saving rate in the CD case is either increasing, decreasing or constant throughout the *entire* transition path. It is clear therefore that if we wish to jointly generate stylized facts [3] and [4], we need to utilize a production function that is both more general than the Cobb Douglas and a production function that can generate non-monotonic transitional paths for the saving rate. It is shown that the CES production function can do just that—and, in the case when the factor substitution elasticity is less than unity, the standard model can theoretically jointly explain all of stylized facts [1] - [4].

It turns out however that the simulation results for the overshooting case do not support the ability of the CES-enhanced standard growth model to explain the post-war decrease in the saving rate. Although the model can generate an overshooting saving rate, the amount of overshooting is not enough to explain the decline in the post-war saving rate.

In sum, if the standard growth model is to explain the saving rate dynamics in the US during the past century or so, the model will have to do so with a choice of technology even more general than CES. More likely however the pure Ricardian nature of the standard model will have to be abandoned in order to explain the recent decrease in the saving rate.

2. The Standard Growth Model With and Without Human Capital

This section briefly outlines the standard growth model with and without human capital. Define:

c_t	=	consumption per labor input at time t.
$\hat{c}_t$	=	consumption per *effective* labor input at time $t = c_t e^{-xt}$.

$f(\hat{k}_t)$	=	output per *effective* labor unit gross of depreciation at time t.
k_t	=	capital stock per labor input at time t.
$\hat{k}_t$	=	capital stock per *effective* labor input at time $t = k_t e^{-xt}$.
n	=	population growth rate.
r_t	=	pre-tax interest rate at time t.
s_t	=	gross saving rate at time $t = 1 - \hat{c}_t/f(\hat{k}_t)$.
s_t^n	=	net saving rate at time $t = s_t - \delta \cdot \hat{k}_t/f(\hat{k}_t)$
$u(\cdot)$	=	isoelastic utility (felicity) function, $u(c) = (c^{1-\theta}-1)/(1-\theta)$.
x	=	rate of labor-augmenting technological change.
z_t	=	share of income consumed at time $t = 1-s_t = \hat{c}_t/f(\hat{k}_t)$.
α	=	capital weight in production, $0 < \alpha < 1$.
δ	=	constant rate at which the capital stock depreciates.
$\gamma_{Z(t)}$	=	growth rate in the share of income consumed at time $t = (\partial z_t/\partial t)/z_t$.
μ_t	=	gross capital share at time $t = \alpha$ for Cobb-Douglas production (σ_{KL}=1).
ρ	=	rate of time preference.
Ψ_T	=	$\log[f(\hat{k}_t)/f(\hat{k}_0)]/\log[f(k_t)/f(k_0)]$ = fraction of cumulative growth in per-capita output between period 0 and t due to transitional dynamics.
Ψ_X	=	$x \cdot t/\log[y_t/y_0]$ = fraction of growth due to technological change = $1-\Psi_T$.
σ_{KL}	=	elasticity of substitution between capital and labor.
θ	=	inverse of the intertemporal elasticity of substitution in consumption.

Households face the following familiar dynamic programming problem first proposed by Frank Ramsey (1928) that can be interpreted á la Robert Barro (1974) as implying operative intergenerational linkages.

(1) $$\max \int_0^\infty u(c_t) \cdot e^{nt} \cdot e^{-\rho t} dt,$$

(with $\rho > n$) subject to the usual dynamic budget constraint and the no-Ponzi game constraint. Production takes the CES form:

(2) $$f(\hat{k}_t) = \left[\alpha \hat{k}_t^{\,1-\frac{1}{\sigma_{KL}}} + (1-\alpha)\right]^{\frac{1}{1-\frac{1}{\sigma_{KL}}}}$$

(It can be shown that the simpler Cobb-Douglas production representation, $f(\hat{k}_t) = \hat{k}_t^{\alpha}$, is a special case of (2) when $\sigma_{KL} = 1$.) As Barro and Sala-i-Martin (1995) argue, the capital share can correspond to a measure of capital broader than physical capital: it can also include human capital. For the Cobb-Douglas case, Barro and Sala-i-Martin choose $\alpha = 0.75$ since this value renders reasonable transitional properties. Denoting the limiting values of variables with asterisks, it can be shown that the limiting values for the interest rate and the saving rate on the balanced growth path are

$$(3) \qquad r^* = \rho + \theta x = f'(\hat{k}^*) - \delta$$

$$(4) \qquad s^* = (x + n + \delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\sigma_{KL}} .$$

The growth rate in the share of income consumed at time t is proven in the Appendix to equal

$$(5) \qquad \gamma_{z(t)} = f'(\hat{k}_t) \cdot \left[z_t - \frac{\theta - 1}{\theta}\right] + \left[s^* \left(\frac{f'(\hat{k}_t)}{f'(\hat{k}^*)}\right)^{1-\sigma_{KL}} - \frac{1}{\theta}\right](r^* + \delta) .$$

3. The Endogenous Saving Rate Along the Transition Path

The next proposition, proven in the Appendix, utilizes equation (14) to characterize the endogenous transitional saving rate for the Ramsey-Barro economy with CES technology.

PROPOSITION 1 *Assume that* $\mathrm{f}(\hat{k})$ *is the CES production function and that* $\hat{k}_1 < \hat{k}^*$. *Then*

(A) If $\sigma_{KL} < 1$ *and the value of* θ *is such that* $s^* \left(\frac{f'(\hat{k}_1)}{f'(\hat{k}^*)}\right)^{1-\sigma_{KL}} \leq \frac{1}{\theta}$, *then the saving rate is decreasing along the transition path from* $\hat{k}_1$ *to* $\hat{k}^*$.

(B) If $\sigma_{KL} > 1$ *and the value of* θ *is such that* $s^* \left(\frac{f'(\hat{k}_1)}{f'(\hat{k}^*)}\right)^{1-\sigma_{KL}} \geq \frac{1}{\theta}$, *then the saving rate is increasing along the transition path from* $\hat{k}_1$ *to* $\hat{k}^*$.

(C) If $\sigma_{KL} = 1$ *and* $s^* = \frac{1}{\theta}$, *then the saving rate is constant along the transition path from* $\hat{k}_0$ *to* $\hat{k}^*$. *(Barro and Sala-i-Martin [1995])*

Cases (*A*), (*B*) and (*C*) nest the possible equilibria corresponding to the Cobb-Douglas (CD) production function ($\sigma_{KL} = 1$) considered already by Barro and Sala-i-Martin (pp. 89-90). Specifically, in the CD case, the inequalities present in cases (*A*), (*B*) and (*C*) reduce to very simple mathematical formulations completely absent of the capital-labor ratio, k: (*A*) $s^* < 1/\theta$, (*B*) $s^* > 1/\theta$ and (*C*) $s^* = 1/\theta$, respectively. The absence of the capital-labor ratio in these formulations reflect the fact that the transitional endogenous saving rate is monotonically (*A*) decreasing, (*B*) increasing or (*C*) unchanging, respectively, throughout the *entire* transition path. It follows that we need a production function that is more general than CD if we want to jointly explain stylized facts [3] and [4].

As summarized in the next proposition, the endogenous transitional saving rate is not necessarily monotonic for CES production. Indeed, the saving rate can even overshoot (undershoot) its long-run value when the factor substitution elasticity is less (greater) than unity.

PROPOSITION 2 *Assume that* $f(\hat{k})$ *is the CES production function and that* $\hat{k}_0 < \hat{k}_1 < \hat{k}^*$ *where* $\hat{k}_0$ *is the initial capital-labor ratio. Then*

(A) If $\sigma_{KL} < 1$ *then the saving rate may increase—possibly overshooting its limiting value—and then will decrease monotonically to* s^* *after a critical point* $\hat{k}_1$ *defined in Proposition 1.*

(B) If $\sigma_{KL} > 1$*, the saving rate may decrease—possibly undershooting its limiting value—and then will increase monotonically to* s^* *after a critical point* $\hat{k}_1$ *defined in Proposition 2.*

(C) If $\sigma_{KL} = 1$*, the saving rate converges monotonically to its long-run value. (Barro and Sala-i-Martin [1995])*

The policy function corresponding to the overshooting case ($\sigma_{KL} < 1.0$) is drawn in the phase diagram in Figure 3. Also drawn in Figure 3 is the policy function corresponding to the constant saving rate case ($\sigma_{KL} = 1.0$ and $s_t = s^* = 1/\theta$): $\hat{c}(\hat{k}) = (1 - s^*) f(\hat{k})$. Both policy functions share the same value of s^*. For Cobb-Douglas production (not drawn), the policy function would never cross the constant saving rate policy function; this non-crossing property reflects the monotonic nature of the transitional saving rate in the CD economy. For CES production however the policy function can cross the constant saving rate policy function as shown in Figure 3, resulting in an overshooting of the saving rate.

4. Simulating the Overshooting Case

This section presents precise calculations (not linear approximations) of the overshooting case noted above. The computations of the policy function were performed using standard numerical techniques (see the Appendix). Following King and Rebelo (1993), I assume that per-capita income increased seven fold during the past 100 years. Moreover, I follow King and Rebelo in my use of the index variables, Ψ_T and Ψ_X, that describe the fraction of growth in per-capita output between period 0 (100 years ago) and time t that is attributed to transitional dynamics and technological change, respectively. For a given value of Ψ_X, the implied rate of technological change equals

$$x = \frac{\Psi_X \cdot \log 7}{100} \tag{6}$$

Moreover, the value of $\hat{k}_0$ (capital-labor ratio 100 years ago) is given by

$$f(\hat{k}_0) = \frac{f(\hat{k}^*_{100})}{e^{\Psi_T \cdot \log 7}} \tag{7}$$

where, to reduce the amount of notation, I assume, without any impact to the numerical calculations, that year 100 (today) "begins" the new steady state, i.e., $f(\hat{k}_{100}) = f(\hat{k}^*)$.

As the earlier propositions suggest, overshooting works best for values of the factor substitution elasticity σ_{KL} that are sufficiently less than unity. It turns out however that choosing a value for σ_{KL} below 0.8 generates an implausibly large value for the interest rate r_0 at time 0, representing 100 hundred years ago (Table 1). This result is true even for an implausibly large value for the capital weight α as well as for an implausibly small value of Ψ_T.[6] (This result is robust to a wide range of values for the other parameters as well; additional tables are available from the author.) Hence, if the transitional dynamics are to explain the post-war decline in the saving rate, they must do so within a fairly narrow range for σ_{KL}, [0.8, 1.0].

In simulating the transition path, an important decision—as Barro and Sala-i-Martin point out—must be made regarding the value of s^* relative to $1/\theta$. I first consider the case in which the two values are equal followed by case in which $s^* > 1/\theta$.

Simulating the Path Between Time 0 and Steady State — Calibration Attempt 1: $s^* = 1/\theta$

The selection of $\sigma_{KL} < 1$ and $s^* = 1/\theta$ implies that $\hat{k}_1$ in Proposition 1 equals $\hat{k}^*$ which, in turn, ensures that the equilibrium saving rate must decrease during at least during some point toward the end of the transition path. Choosing a reasonably large value of Ψ_T also helps to generate an overshooting path since the larger the value of Ψ_T, the more transitional dynamics will take place.

Moreover, the larger the value of Ψ_T, the larger the value of r_0 relative to r^* and hence the more likely that the saving rate will be increasing during at least some part early into the transition path as the representative consumer's income effect will tend to dominate the consumer's substitution effect. Later along the transition path, the substitution effect will tend to dominate, reversing the sign of the saving rate derivative.

Unfortunately, my simulations (not reported) clearly show that none of these parameter choices—even though favorable for generating an overshooting saving rate path—actually generate an overshooting saving rate when $s^* = 1/\theta$. This is true even for very small values for σ_{KL} (corresponding to implausibly large initial interest rates) as well as for large values for Ψ_T and large values for r^* (a larger value of r^* affords a larger value of θ without violating the constraint $\rho > n$.) These results were robust to considerable variation in the capital-output ratio and to other parameter choices as well. The reason for the difficulty in generating an overshooting transition path is that the choice of $s^* = 1/\theta$ tends to numerically constrain the transition path by evidently enough of an extent such that the winner between the competing income and substitution effects near the steady state gets reflected back all the way to the beginning of the transition path at $\hat{k}_0$. Hence the saving rate is consistently monotonically decreasing throughout the entire transition path when $s^* = 1/\theta$.

Simulating the Path Between Time 0 and Steady State — Calibration Attempt 2: $s^* > 1/\theta$

Assuming $s^* > 1/\theta$ allows for a greater amount of variation of the gross saving rate early into the transition path by, conceptually speaking, pushing the point $\hat{k}_1$ further along the transition path compared to the case $s^* = 1/\theta$. On the other hand, assuming that s^* is too much bigger than $1/\theta$ means that we step further away from the sufficient conditions of the previous propositions which implies that we are no longer guaranteed that the endogenous saving rate will decline prior to the steady state. A delicate balancing act therefore is required.

Figures 4 and 5 present the transition paths for $\sigma_{KL} < 1$ and for clever choices of $s^* > 1/\theta$ that—after numerous guesses—balance the competing effects noted in the previous paragraph. Figure 4 employs $\sigma_{KL} = 0.60$ and a conventional capital-output ratio of 2.8 whereas Figure 5 employs $\sigma_{KL} = 0.80$ and a larger capital-output ratio equal to 4.5 that reflects a broader measure of the capital stock including at least some human capital. Notice that the transition path for the gross saving rate does indeed overshoot its limiting value in both figures. Not surprisingly, the results in Figure 4, that employ $\sigma_{KL} = 0.60$, are not very persuasive due to the large value of r_0 as well as due to the counterfactual variation in both the conventional measures of the capital share and capital-output ratio, and the quick convergence rate (half-life = 18 years). Figure 5 however is a little more plausible. The value of r_0 is only 0.19 (with $r^* = 0.06$) and the variation in the gross capital share across the entire transition path is minimal (from around 0.68 to around 0.60). The capital-output ratio increases from 2.5 to 4.5 which is reasonable, especially under a broad interpretation of the capital stock that includes human capital. The half-life for income convergence is a respectable 22 years although a more persuasive half-life would be around 35 to 40 years.

Figures 4 and 5 show that, although the overshooting phenomenon associated with the parameterization $\sigma_{KL} < 1$ can span numerous decades, the actual magnitude of the overshooting of the saving rate—measured as the difference between the peak of the transitional saving rate and its limiting value—is only about a few hundredths of a percentage point for a realistic parameterization of the model. This small magnitude of overshooting presents a rather convincing case against the ability of the standard growth model with CES technology in explaining the recent decrease in the endogenous gross saving rate in the US during the past two decades.

5. Conclusions

This paper took the standard growth model seriously and examined whether the model can explain the large post-war decline in the savings rate in the US and other G-7 countries in light of its previous pre-war non-decrease and in the context of two other stylized facts. The paper first reviewed and rejected the more simple explanations of the decrease in the national saving rate compatible with the standard model. The paper then argued that the best hope for the standard model lied in explaining the changes in the saving rate as a result of transitional dynamics—but that doing so required a production function that was more general than Cobb Douglas.

The properties of the transitional endogenous saving rate were then derived in the context of CES production. It was then proven that the gross saving rate might overshoot (undershoot) its limiting value when the factor substitution elasticity is less than (exceeds) unity. The overshooting case offers a possible explanation of the saving rate dynamics during the previous century.

Exact numerical calculations of the transition path showed that employing a value of the substitution elasticity between 0.8 and 1.0 was able to generate only a very small amount of overshooting—far from the overshooting empirically observed in the time series data. Somewhat surprisingly, this result was true even when using an elasticity equal to 0.60 corresponding to implausible initial conditions and an implausible transition path. Although I simulated only the US economy, the results are robust to such a large parameter range and assumptions that the general results should apply to all of the G-7 countries with saving behaviors similar to that of the US.

Of course, it is possible that one might some day find a production function that generates the post-war decline in the saving rate in light of the other stylized facts typifying the US growth process during the past century. Nonetheless, the standard model with a flexible production function (CES) does not do the job. But a modified model in which at least a fraction of the agents are non-Ricardian *is* able to generate a sizeable short-to-medium run decline in the post-war saving rate in the presence of the post-war growth in unfunded liabilities. This is true in a model in which Ricardian and lifecycle agents co-exist and it is also true in the Blanchard (1985) model which can be interpreted as incorporating probabilistic altruistic linkages (Blanchard [1985, footnote 1]).[7]

Appendix

DERIVATION OF EQUATION (5)

The "equations of motion" for the model in Section 2 can be shown to be:

(A.1) $$\frac{\partial \hat{c}_t/\partial t}{\hat{c}_t} = \frac{1}{\theta}[r_t - \rho - \theta x]$$

(A.2) $$\partial \hat{k}_t/\partial t = f(\hat{k}_t) - \hat{c}_t - (x + n + \delta)\hat{k}_t$$

It is also easy to demonstrate that the following relationship holds between the interest rate and the capital-output ratio at time t for CES production:

(A.3) $$\left(\frac{\alpha}{r_t + \delta}\right)^{\sigma_{KL}} = \frac{\hat{k}_t}{f(\hat{k}_t)}$$

By definition,

(A.4) $$\gamma_{z(t)} = \frac{\dot{z}_t}{z_t} = \frac{\partial \hat{c}_t/\partial t}{\hat{c}} - \frac{f'(\hat{k}_t)\cdot \partial \hat{k}_t/\partial t}{f(\hat{k}_t)}$$

Substitute equations (A.1), (A.2), (A.3), (3) and the definition of z_t into (A.4) to get:

(A.5) $$\gamma_{z(t)} = \frac{1}{\theta}\Big[f'(\hat{k}_t) - \delta - \rho - \theta x\Big] - (1-z_t)\cdot f'(\hat{k}_t) + \alpha^{\sigma_{KL}} f'(\hat{k}_t)^{1-\sigma_{KL}}(x+n+\delta)$$

Now solve equation (4) for $(x + n + \delta)$ and substitute into equation (A.5). Re-arrange to get,

(A.6) $$\gamma_{z(t)} = f'(\hat{k}_t)\left[\frac{1}{\theta} - (1-z_t)\right] + \left\{s^*\left[\frac{f'(\hat{k}_t)}{\rho+\theta x+\delta}\right]^{1-\sigma_{KL}} - \frac{1}{\theta}\right\}(\rho + \theta x + \delta)$$

Equation (5) comes from a simple rearrangement of the first term in (A.6) and from substituting equations (15) and (3) into the second term of (A.3).■

PROOF OF PROPOSITION 1.[1]

Consider case (A). Note that $\sigma_{KL} < 1$ implies that $\left(\frac{f'(\hat{k}_1)}{f'(\hat{k}^*)}\right)^{1-\sigma_{KL}} > 1$ since $\hat{k}_1 < \hat{k}^*$. Hence,

[1] This proof generalizes the derivation of Barro and Sala-i-Martin (1995, Appendix 2B), who considered Cobb Douglas technology, to the case of CES technology.

$s^* < 1/\theta$. Now suppose that there is a value of t such that $z_t \le \frac{\theta - 1}{\theta}$. Equation (5) then implies that $\gamma_{Z(t)} < 0 \Rightarrow z_{t+1} < z_t \Rightarrow \gamma_{Z(t+1)} < 0 \Rightarrow z_{t+2} < z_{t+1}$, *ad infinitum*. Hence, $z_t \to 0 \Rightarrow s_t \to 1$ which contradicts $s^* < 1/\theta$. It follows that $z_t > \frac{\theta - 1}{\theta}$ $\forall\, t > 0$. Now differentiate (5) with respect to t:

$$\text{(A.7)}\qquad \begin{aligned} \dot{\gamma}_{z(t)} &= f''(\hat{k}_t) \cdot \frac{\partial \hat{k}_t}{\partial t} \cdot \left[z_t - \frac{\theta - 1}{\theta} \right] + f'(\hat{k}_t) \cdot \gamma_{z(t)} \cdot z_t \\ &\quad + s^*(r^* + \delta)(1 - \sigma_{KL}) \left(\frac{f'(\hat{k}_t)}{f'(\hat{k}^*)} \right)^{-\sigma_{KL}} \frac{f''(\hat{k}_t)}{f'(\hat{k}^*)} \cdot \frac{\partial \hat{k}_t}{\partial t} \end{aligned}$$

Equation (A.7) implies that $\gamma_{Z(t)} > 0$ $\forall\, t > 0$. To prove this claim, suppose the that there is a value of t such that $\gamma_{Z(t)} \le 0$. Then the first and third terms on the RHS of equation (A.7) are both negative and the second term is weakly negative, which implies that $\dot{\gamma}_{z(t)} < 0 \Rightarrow \gamma_{Z(t+1)} < \gamma_{Z(t)} \le 0 \Rightarrow \dot{\gamma}_{z(t+1)} < 0 \Rightarrow \gamma_{Z(t+2)} < \gamma_{Z(t+1)}$, *ad infinitum*. Hence, $z_t \to 0$ which contradicts $z_t > \frac{\theta - 1}{\theta}$. It follows that $\gamma_{Z(t)} > 0$ $\forall\, t > 0$ and therefore $\frac{\partial s_t}{\partial t} < 0$. The other cases can be proven in a similar fashion. Note that for case (*B*), most of the inequalities and signs in the above formulae are reversed. ■

PROOF OF PROPOSITION 2.

The first part of the proposition for case $\sigma_{KL} < 1$ ("*the endogenous saving rate may overshoot its limiting value*") is proven by example in the text. The second half of the proposition follows directly from Proposition 1. Similar is true for the case $\sigma_{KL} > 1$. Case $\sigma_{KL} = 1$ is proven in the text.■

SOLVING THE POLICY FUNCTION

The policy function for the model herein can be described implicitly as the solution to the following differential equation:

$$\text{(A.8)}\qquad \frac{\partial \hat{c}(\hat{k})}{d\hat{k}} = \frac{\hat{c}(\hat{k})\left[f'(\hat{k}) - \delta - \rho - \theta x\right]}{f(\hat{k}) - \hat{c}(\hat{k}) - (x + n + \delta)\hat{k}} \qquad ,$$

with the boundary (or initial) values given by $(\hat{c}, \hat{k}) = (\hat{c}^*, \hat{k}^*)$. To see this, note that $\frac{d\hat{c}}{d\hat{k}} = \frac{\partial \hat{c}(\hat{k}_t)/\partial t}{\partial \hat{k}_t / \partial t}$

where the numerator and the denominator are given by equations (A.1) and (A.2), respectively. Equation (A.8) is solved numerically for the policy function using the *shooting method* or the *time-elimination method*. The standard *shooting method* employs the bisection updating technique to solve for the correct value of $\hat{c}_0$ such that the generated policy function satisfies the transversality condition. The *time-elimination method* outlined recently by Casey Mulligan and Xavier Sala-i-Martin (1993). This novel approach transforms the standard boundary-value problem into an easier initial-value problem by employing L'Hôpital's rule at the limit point $(\hat{c},\hat{k}) = (\hat{c}^*,\hat{k}^*)$ (the slope of the policy function, equation (A.8), is indeterminant at the limit point). Applying L'Hôpital's rule to equation (A.8) at the limit point renders a quadratic equation in $\partial\hat{c}(\hat{k}^*)/d\hat{k}$. The positive root of this quadratic equation is the correct solution since $\partial\hat{c}(\hat{k}^*)/d\hat{k} > 0$ (Figure 3). After a little algebra, it can be shown that the positive root in the model herein is given by

(i) $$\frac{\partial\hat{c}(\hat{k}^*)}{d\hat{k}} = \frac{\left[f'(\hat{k}^*) - (x+n+\delta)\right] + \left\{\left[f'(\hat{k}^*) - (x+n+\delta)\right]^2 - 4\frac{\hat{c}^*}{\theta}f''(\hat{k}^*)\right\}^{1/2}}{2}.$$

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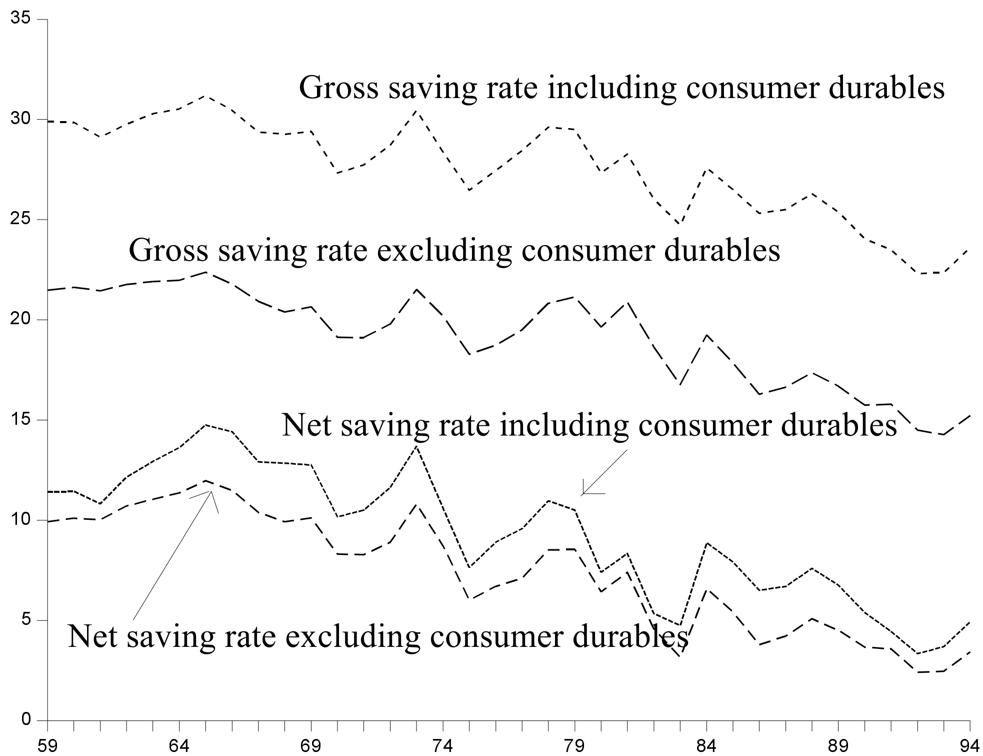
The US Gross and Net National Saving Rates in the Post-WWII Era
(Figure 1)

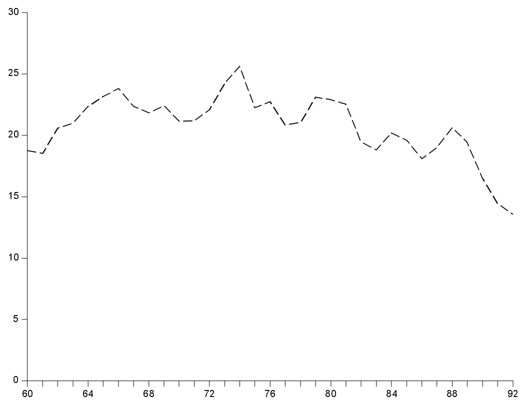


Sources / Notes: (1) Calcualations of the gross national saving rates are consistently measured and based on the newly revised NIPA accounts which include all form of government saving (including military investment).

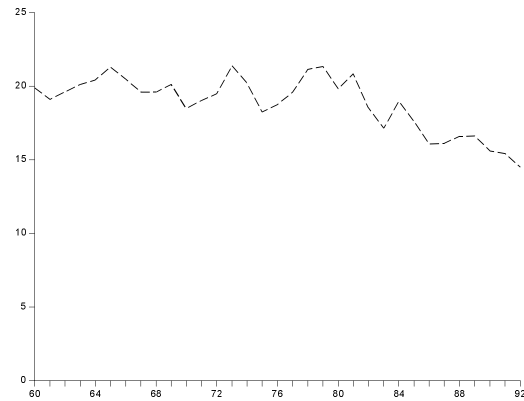
(2) The net saving rates equal the gross saving rates less capital consumption, as measured in the Fed's Flow of Funds.

The Gross and Net National Saving Rates for the G-7 in the Post-WWII Era
(Figures 2 [*a* - *g*])

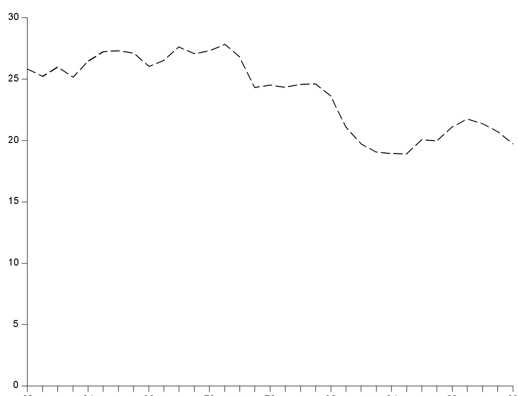
(*a*) Canada



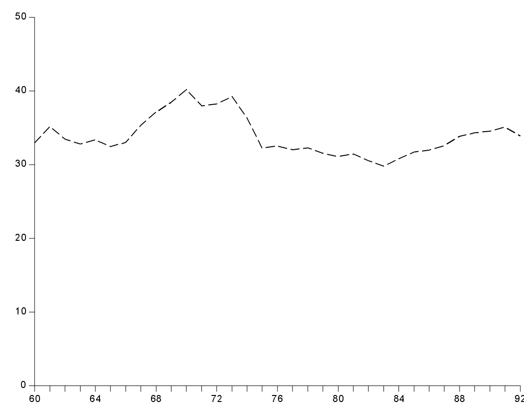
(*b*) United States



(*c*) France

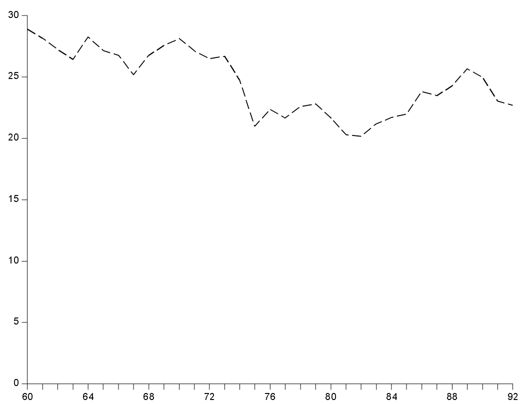


(*d*) Japan

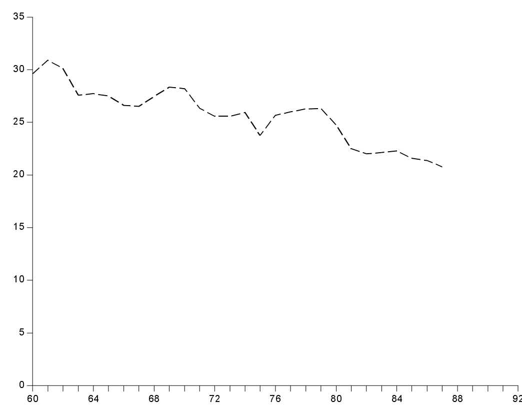


The Gross and Net National Saving Rates for the G-7 in the Post-WWII Era, Continued
(Figure 2, Continued)

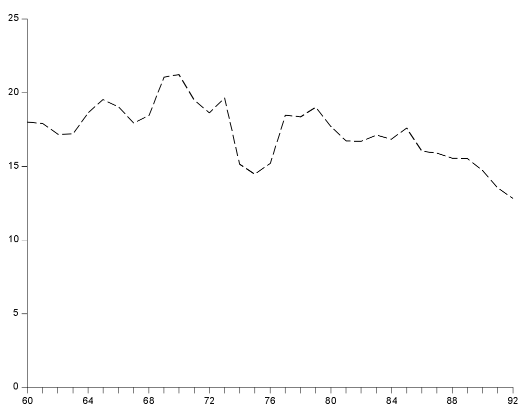
(*e*) Germany



(*f*) Italy



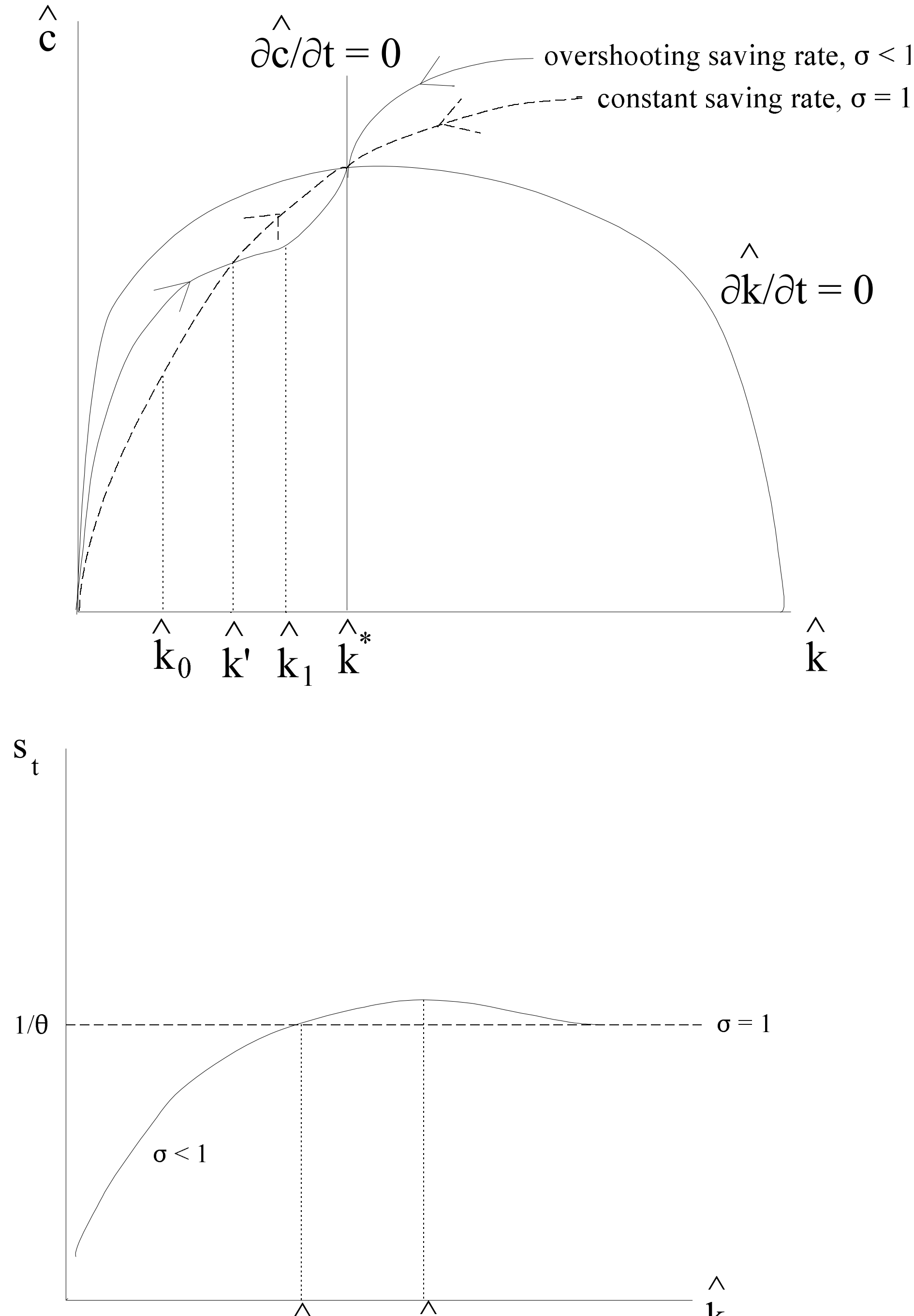
(*g*) United Kingdom



Sources / Notes: (1) Gross national saving rates computed from data supplied by the OECD and follows the UN definition of gross saving which excludes both consumer durables and some of government saving (investment in military capital).
(2) Data for Italy from 1988 - 1992 was not available.

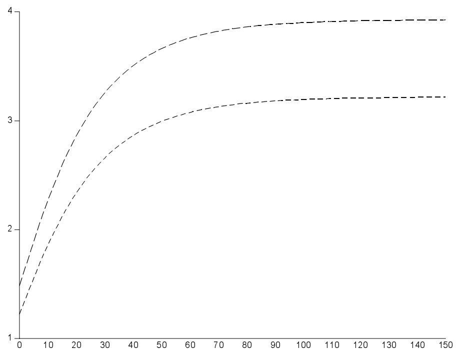
Phase Diagram for the Neoclassical Production Function with CES Production and $\sigma_{KL} < 1.0$ and the Implied Saving Rate: The Case of an Overshooting Saving Rate

(Figure 3)

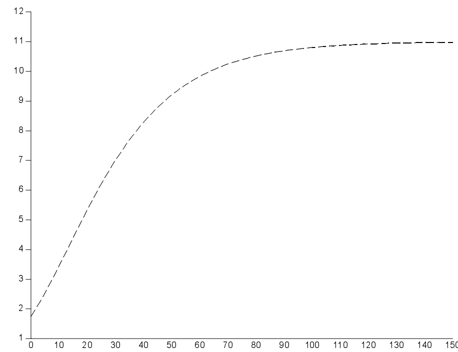


The Transition Path for Parameterization Noted Below
(Figures 4 [*a* - *f*])

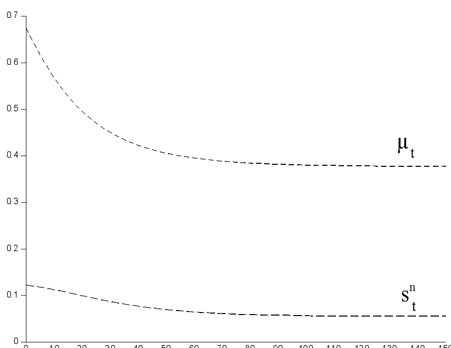
(*a*) $\boldsymbol{f(\hat{k}_t)\,,\,\hat{c}_t}$



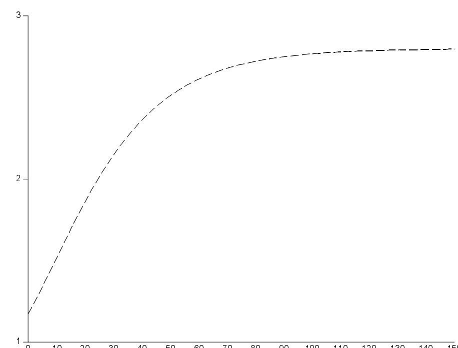
(*b*) $\boldsymbol{\hat{k}_t}$



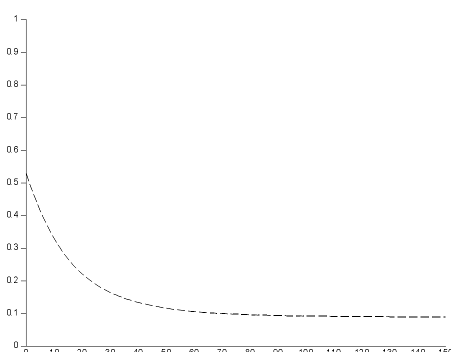
(*c*) $\boldsymbol{\mu_t\,,\,s_t\,,\,s_t^{\,n}}$



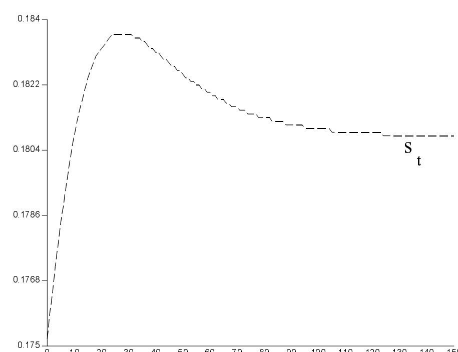
(*d*) $\boldsymbol{\hat{k}_t/f(\hat{k}_t)}$



(*e*) $\boldsymbol{r\hat{c}_t}$



(*f*) $\boldsymbol{s_t\,,\,s_t^{\,n}}$

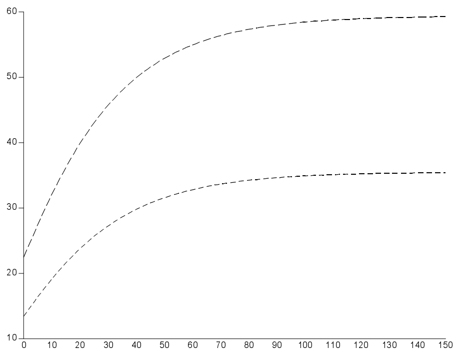


Parameter Vector: $r^*=0.09$, $\hat{k}^*/f(\hat{k}^*)=2.8$, $\sigma_{KL}=0.60$, $\Psi_T=0.50$, $\alpha = 0.75$, $\theta = 8.0$, $n = 0.01$

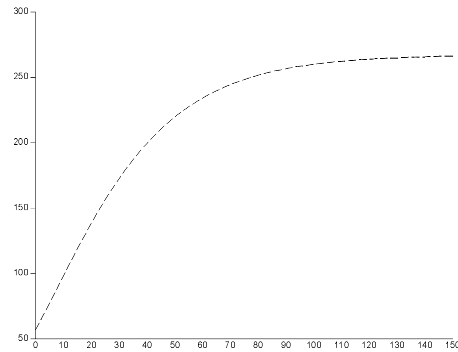
Implied Values: $\delta = 0.045$, $\rho = 0.012$, $x = 0.01$, half-life of output = 18 years.

The Transition Path for Parameterization Noted Below
(Figures 5 [*a* - *f*])

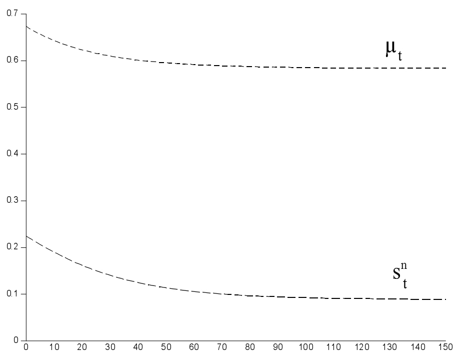
(*a*) $f(\hat{k}_t)\,,\hat{c}_t$



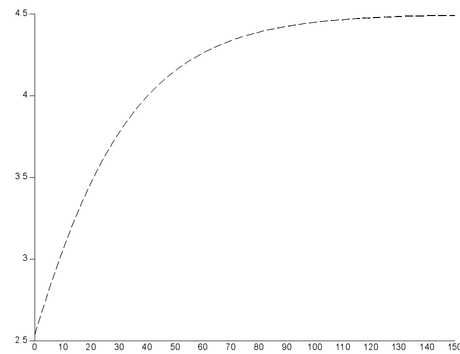
(*b*) $\hat{k}_t$



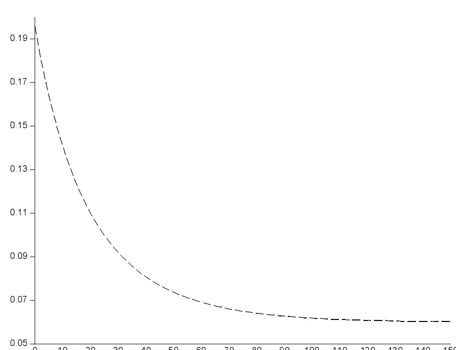
(*c*) $\mu_t\,, s_t\,, s_t^{\,n}$



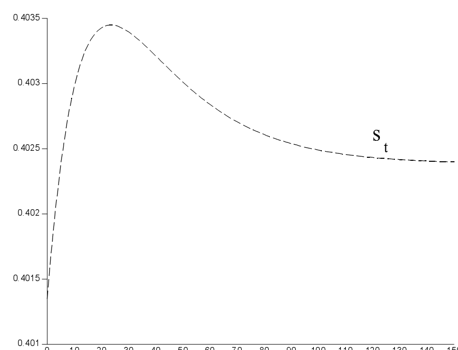
(*d*) $\hat{k}_t/f(\hat{k}_t)$



(*e*) $r\hat{c}_t$



(*f*) $s_t\,, s_t^{\,n}$



Parameter Vector: r^*=0.06, $\hat{k}^*/f(\hat{k}^*)$=4.5, σ_{KL}=0.85, Ψ_T =0.50, α = 0.75, θ = 3.0, n = 0.01
Implied Values: δ = 0.07, ρ = 0.031, x = 0.01, half-life of output = 22 years.

Endnotes for Referee
(Not necessary to print unless desired by referee or editor)

1. In the 1980s, some researchers initially dismissed the post-war decline in the saving as a measurement problem. See, for example, Patric Hendershott and Joe Peek (1987). More recent research (and apparently the consensus opinion) contradicts this claim. See, for example, Robert Lipsey and Irving Kravis (1987), Lans Bovenberg and Owen Evans (1990), the 1993 CBO Study *Assessing the Decline in the National Saving Rate* and Martin Browning and Annamaria Lusardi (1996). Indeed, using the usual definitions, personal, private and public saving has decreased in most G-7 countries, including the US, since the end of WWII. These results are robust to (*a*) changes in depreciation assumptions (notice that the gross saving rate in Figure 1 has also declined), (*b*) counting consumer durables as saving (Figure 1), and (*c*) including changes in the value of assets (CBO [1993]; also see James Poterba and Andrew Samwick [1995]). Looking at (*d*) a broader measure of saving to include human capital investment only reenforces the decline. The data suggests that investment in education (particularly at the secondary level) expanded enormously from 1910 to 1960 (Claudia Goldin [1994]) during the time in which saving in physical capital also (slightly) increased. The investment of human capital (especially college enrollment) dramatically reversed course however in the 1970s after several decades of historic growth (Kevin Murphy and Finis Welch [1989]), during the time in which investment in physical capital was also declining. Both Dale Jorgenson and Barbara Fraumeni (1989) and CBO (1993) reach this same conclusion. (*e*) Spending on research and development and education services also declined in the 1980s and so including them would also only reenforces the picture (CBO [1993]).

2. The calculations conservatively assume a gross capital share equal to 0.40, 1.0 for the factor substitution elasticity, 0.06 for the depreciation rate, 0.01 for the rate of labor-augmenting technological change, 0.01 for the population growth rate and an initial gross saving rate equal to 0.25 (the current actual value inclusive of consumer durables). These values imply a value for the initial interest rate equal to 0.068. Alternative plausible parameter values (such as excluding consumer durables) generate even bigger gains.

3. For the specific case of the US, economic historians have believed since Simon Kuznets' seminal work (see, e.g., Simon Kuznets [1966]) that the saving rate in the US was either unchanged or, more likely, somewhat higher in 1950 than it was a century earlier. This stylized fact appears to be robust to a rather wide variation of definitions of the saving rate. For the other G-7 countries, the saving rate has increased, typically considerably, during this same time period. See Lance Davis and Robert Gallman (1973, 1978), Robert Gallman (1966), Angus Maddison (1992) and Ian McLean (1994) for historic evidence of saving rates in the US and the other G-7 nations.

4. David Cutler, James Poterba and Louise Sheiner and Lawrence Summers (1990) conclude that demographic factors can be behind the post-war decline in the saving rate but concede that the decline in the saving rate is "greater than what our analysis suggests can be justified

by demographic factors" alone (p. 54). Moreover, more recent work has downplayed the importance of demographic effects even more or produced the opposite sign. See, for example, Barry Bosworth, Gary Burtless and John Sabelhaus (1991), James Poterba (1991), and Lans Bovenberg and Owen Evans (1990).

5. Following the passage of the 16th Amendment in 1913, effective tax rates on capital income increased from near zero percent to around 50% by the early 1950s (Jane Gravelle [1994]), during the same period that the saving rate was either constant or slightly increasing. Over the past four decades however these tax rates declined (*ibid*), during the same period that the saving rate was decreasing. Of course, these correlations are not conclusive since, for example, a lower tax rate may be chosen in response to a decrease in saving, although this would indicate that the decrease in the saving rate is coming from another source besides taxes.

6. Theoretically, a large value of α should dampen the marginal product of capital at time 0 and hence should tend to reduce the initial interest rate, r_0. The smaller the value of Ψ_T, the smaller the difference between the initial value of the effective capital-labor ratio $\hat{k}_0$ and its final value $\hat{k}_0^*$— and so the closer the value of r_0 is to r^* (i.e., $\Psi_T \downarrow \Rightarrow \hat{k}_0 / \hat{k}^* \downarrow \Rightarrow \downarrow$).

7. In Smetters (1995), I simulated the post-war growth in the (mostly unfunded) Social Security program using a heterogenous-agent model in which one type is a pure lifecycle agent who lives for 55 years (as in Auerbach and Kotlikoff [1987]) and the other agent type, who also lives for 55 years, has altruistic linkages toward their children. The presence of altruistic agents implies that the long-run interest rate always returns to its initial steady-state value after a change in the timing of taxes. However, in the short run—the half life of which can last up to a century—the saving rate differs significantly from its long-run value. A significant short-run reduction in the saving rate following a change in the timing of taxes can also be found using the Blanchard (1985) model (Hamid Faruqee, Douglas Laxton, and Steve Symansky [1996]).

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