Understanding Adverse Selection in the Annuities Market and the Impact of Privatizing Social Security

by

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Abstract

Few Americans purchase life annuities as protection against life span uncertainty. This fact has often been attributed to adverse selection: individuals who expect to live longer are more likely to purchase annuities and this behavior forces insurance companies to charge higher premia than implied by average survival probabilities. Indeed, an empirical study by Mitchell, Poterba and Warshawsky (1997) finds that adverse selection raises annuity prices by 8 to 10 percent above actuarially fair prices based on average mortality.

A fundamental question still unanswered in the literature is whether the observable price increase caused by adverse selection can also be generated in a realistic life cycle model. A related second question is how much of the adverse selection problem can be attributed to the existence of social security. This paper calibrates a pure life cycle model for a characteristic cohort of the US economy and endogenously derives the degree of adverse selection. The variation in survival probabilities is based on income, race, and marital status. The model can reproduce three stylized facts: First, it generates annuity premia for 65 year males that exceed those based on average survival probabilities by 7 to 10 percent. Second, adverse selection increases with the age of the annuitant. Third, the adverse selection problem is smaller in the annuities market for females than in the market for males.

Although eliminating social security may reduce the adverse selection induced increase in annuity prices by 1 to 2.5 percentage points, it cannot remove adverse selection. Moreover, a significant share of the measured adverse selection is attributable to the positive correlation between longevity and income and cannot be eliminated with a mandatory annuity that increases with the income of the annuitant.

"In life, one sometimes makes bad deals."

Comment by now 122 year old Jeanne Louise Calment on the notary public Andre-Francois Raffray who purchased her apartment for a monthly annuity of $500 when Calment was 90 years old. Raffray paid twice the market value for the apartment and died in December at age 77. His widow and children are obligated to continue the payments until Calment's death.

1. Introduction

Few Americans purchase life annuities as protection against life span uncertainty. The weak demand for private annuities has been largely attributed to two effects: unfavorable annuity prices due to adverse selection, and crowding out of annuity demand by Social Security and private pensions. Adverse selection arises if the insurance company cannot distinguish among different risk types. In the case of annuity insurance the insurance company may have less information about longevity prospects than the potential annuitant. Since consumers who expect to live longer -- the high risk type for the insurance company -- buy larger annuities, the insurance company incurs a loss if it charges a premium based on average survival probabilities. Consequently, prices for annuities rise above those based on the average population mortality, and higher prices may induce some consumers to drop out of the annuities market.

This paper investigates the price of single-premium immediate life annuities. Under this type of insurance contract, the insurance company converts a lump sum payment into a constant income stream contingent on the survival of the annuitant. Previous papers on single-premium immediate life annuity insurance by Friedman and Warshawsky (1988,1990) have examined observed annuity prices. Based on data for 1968 to 1983, they find that due to adverse selection single life annuities in the US were 13 to 15 percent more expensive than could be expected from average survival probabilities of the population. A recent paper by Mitchell, Poterba, and Warshawsky (1997) applies a similar methodology and finds that in 1995 adverse selection reduced the value of annuities by 8 to 10 percent.

A fundamental question still unanswered in the literature is whether the observable cost of annuities can be endogenously generated in a realistic life cycle model. This paper derives the price of annuities in a 75 period life cycle model with life span uncertainty. The model is calibrated to a cohort of the 1995 Current Population Survey and reflects the empirical variation of survival probabilities with family income and marital status. It also reflects the progressivity of the current Social Security system.

A second and related question is how much of the lack of actuarial fairness can be blamed on the existence of social security, i.e. whether a realistic life cycle model would predict a significant improvement in annuity prices were social security benefits unavailable. Social security provides an annuity in old age and therefore crowds out private demand of annuities. Since social security induces individuals with shorter life expectancy to reduce their annuity demand disproportionately more than individuals with longer life expectancy, social security may exacerbate the adverse selection problem (Abel, 1986). This question of the interaction between social security benefits and private annuity demand has received increasing attention recently due to the debate on reforming the social security. Some reform proposals would force individuals to rely much more on private retirement planning. Therefore, the cost of insurance against life span uncertainty is a crucial criterion for evaluating these proposals.

This paper shows that a life cycle model can generate annuity prices that are 6.5 to 10.5 percent higher than actuarial fair prices for 65-year-old males. The model also reflects the empirical result that adverse selection raises prices in the annuities market for women to a lesser extent than for men. Moreover, the model reproduces the stylized fact that the adverse selection problem rises with the age of the annuitant.

Privatization -- here understood as the elimination -- of social security may reduce the adverse selection induced loading of annuity prices by 1 to 2.5 percentage points for individuals of age 65, a reduction between 15 and 35 percent. Hence, the increasing market size after privatization can considerably lower the impact of adverse selection. However, privatization cannot reduce adverse selection to negligible proportions. Moreover, the simulations reveal that between 20 and 40 percent of the measured adverse selection is due to the positive correlation between income and mortality and cannot be removed by mandating annuitization of private accounts that are proportional to income. This result is important for regulating a privatized social security system, as discussed in Walliser (1997).

The paper is organized as follows. Section 2 discusses previous theoretical and empirical findings concerning adverse selection. Section 3 develops a two period model of annuity demand and derives insurance load factors. Section 4 extends the simulation model to 75 periods (ages 25-100) and calibrates it to a cohort of 50-year-old agents using an empirically based spread in survival probabilities. Section 5 summarizes and concludes the paper.

2. Annuity Insurance and Adverse Selection -- Theory and Evidence

Consumers who seek to smooth consumption over the life cycle have two strong incentives to purchase an annuity. First, an annuity insures against outliving one's resources. If death is uncertain and annuities are unavailable, the consumer may reduce his assets too quickly if she outlives her expected life span. Consumers with shorter than expected lives, on the other hand, may leave a large accidental bequest since at the time of death not all assets are consumed. An annuity can avoid both outcomes by converting assets into a continuous stream of income contingent on survival. Second, an annuity pools the resources of annuitants and can therefore offer a higher rate of return than an alternate non-annuitized asset. Those annuitants who die early implicitly leave their estate to other annuitants rather than to their descendants which increases the rate of return of the annuity.

Ideally insurance companies would have full information and each individual could purchase an annuity based on his specific survival probabilities. Under such a regime, the assets of each consumer would be converted into an income stream that is actuarially fair for each consumer. However, insurance companies generally do not have enough information about survival prospects to calculate actuarially fair premia for individuals. Either information can be successfully hidden by prospective annuitants or regulatory constraints may prevent insurers from making use of available data, for example by forcing them to offer the same contract to males and females.

If individuals know more about their own survival prospects than the insurer, those with a higher chance to live for a long time will buy more annuities. Such behavior may raise annuity prices and may even induce individuals to drop entirely out of the market. This phenomenon has been called adverse selection.

2.1. Empirical Evidence for Adverse Selection

How Well Can People Assess Their Own Survival Prospects?

Adverse selection can only arise if individuals possess private information about their own life expectancy, information that they can successfully hide from any prospective insurer. Empirical evidence from two papers suggests that individuals are capable to assess their own survival prospects based on factors generally unobservable to insurers. This finding supports the idea that there is sufficient private information about longevity to cause adverse selection in annuities markets.

Hamermesh (1985) who conducted a survey of 410 white male economists and 363 white male residents of a Midwestern SMSA, finds that agents' responses are roughly consistent with life tables. Most importantly, subjective probabilities are correlated with forebears' longevity as well as smoking habits and illness. These factors play a role in life expectancy. However, according to Hamermesh, agents rely *more* on this information in predicting their own survival prospects than empirically justified. Respondents are also more optimistic than experience suggests in extrapolating gains in life expectancy.

Recent results from the first wave of the Health and Retirement Survey support Hamermesh's findings. Hurd and McGarry (1995) exploit a question that asked respondents to evaluate their chances to live to age 75 and age 85. They conclude that the implicit subjective probabilities are generally internally consistent and vary systematically with socioeconomic and behavioral variables such as education, income, and smoking. Those variables are good predictors for actual outcomes.[1]

To summarize, the empirical evidence in these two studies suggests that individuals can predict their own survival prospects with some accuracy. Thus, to the extent that the privately available information about forebears' longevity, health habits and other individual characteristics cannot be obtained by insurers for pratical or legal reasons, there is sufficient scope for private information on survival probabilities and adverse selection in the annuities market.

1. It remains to be seen from future waves of the HRS, how well subjective probabilities match actual outcomes in the sample.

The Cost of Private Annuities

A number of important papers by Friedman and Warshawsky (1988, 1990) and Warshawsky (1988) thoroughly analyze annuity prices in the United States.[2] They compile prices for single immediate life annuities charged by the ten largest suppliers of these contracts between 1968 and 1983. A single immediate life annuity converts a one time cash payment into an immediate income stream conditional on survival. In contrast to a deferred annuity it has no investment portion.

Friedman and Warshawsky (1988) find that actual annuity prices are between 24 and 39 percent higher than implied by the value of an actuarially fair annuity if the price of the latter is calculated with population average survival probabilities.[3] However, only part of this so-called 'loading' or 'load factor' is due to adverse selection. Some portion of the price is attributable to overhead costs caused by administrative costs, taxes, and profits.

In order to identify the magnitude of adverse selection, Friedman and Warshawsky (1988, 1990) use the difference in survival probabilities between annuity purchasers and the average population. By comparing the actuarially fair price for the general population with the actuarially fair annuity price for the subpopulation of annuity purchasers, they find that between 13 and 15 percentage points of the load factor can be explained by adverse selection. The remaining loading is due to overhead costs.[4]

Overhead costs identified by Friedman and Warshawsky are more variable than the loading of prices due to adverse selection. This finding suggests that the pool of insurance purchasers is relatively similar across insurance companies and that other factors, possibly transaction costs or firm size, explain the variation in insurance costs. Such an observation supports the assumption of

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2. It should be noted that currently available annuities are nominal annuities and not insured against inflation. Friedman and Warshawsky (1990) take this implicitly into account by discounting with a nominal rate of return in their calculations.

3. I am reporting the results which use the 20 year government bond rate for discounting. The results using the corporate bond rate tend to show similar load factors due to adverse selection but higher overall loading.

4. According to Friedman and Warshawsky (1988) the loading due to overhead costs is comparable to other insurance markets. Since this paper focuses on adverse selection overhead costs are not taken into account.

pooled insurance market.

Annuity prices have been reevaluated with an improved methodology in a recent paper by Mitchell, Poterba and Warshawsky (1997). They correct for the term structure of interest rates and also account for taxes. Furthermore, the change in the pool of annuitants at different ages is explicitly recognized. Mitchell, Poterba, and Warshawsky conclude that, depending on the discount rate, the loading of annuity insurance premia in 1995 amounted to 17 to 25 percent for 65-year-old males. According to their findings, adverse selection accounts for 8 to 10 percentage points of the measured loading. The remaining 9 to 17 percentage points are attributable to overhead costs. Moreover, they find a significant increase in the effect of adverse selection with age and a significantly smaller effect of adverse selection on annuity prices for women.

From a theoretical perspective, Friedman and Warshawsky (1988) conclude that their empirical load factors cannot easily explain the fact that few Americans buy annuities. They suggest that a non-trivial bequest motive combined with mandated annuitization through social security is necessary to explain the empirical observation. However, their 1990 paper shows that this result may change if they consider a constant differential in the rate of return of annuities; a social annuity combined with the observed yield differential may already shut down the market. It seems therefore important to generate the adverse selection endogenously rather than impose an exogenous yield differential to resolve the question whether the small incidence of purchases can be solely explained with adverse selection.

2.2 Adverse Selection Equilibria

Most theoretical papers dealing with adverse selection have applied the Rothschild-Stiglitz (1976) or the Wilson (1977) equilibrium concept.[5] The existence of these equilibria requires that each insurance company can perfectly monitor the insurance purchases of all customers and prevent

5. See, for example, Eckstein, Eichenbaum and Peled (1985).

customers from buying more than one contract.[6] Whereas this assumption may be justifiable for some insurance markets as Rothschild and Stiglitz (1976) argue, it seems difficult to apply to the annuity insurance market. Even if an annuity contract explicitly restricted further purchases of annuities, this might not suffice since monitoring would be very difficult. First, the receipt of annuity payments from different insurance companies would not be observable. Additionally, in contrast to other insurance markets, the catastrophe (death) for the customer ends the liability of the insurance company rather than creating it. This fact precludes withholding payments and investigating compliance with the insurance contract as, for example, in fire insurance or car insurance markets. It also rules out market separation through insurance menus with different copays as for medical insurance.

If insurance companies cannot control the quantity of insurance purchased, it seems reasonable to employ an equilibrium concept in which the insurer can only choose the premium of the annuity. Indeed, this is the kind of equilibrium considered by Abel (1986) and Pauly (1974) that will also be subsequently employed in this paper. Such an equilibrium could be alternatively interpreted as a Nash equilibrium in which any firm deviating from the zero profit premium will incur losses.[7]

The choice of this equilibrium concept is supported by the empirical observation that annuity insurance companies charge a premium per dollar of monthly annuity payments and allow the customer to choose the size of the annuity. Nobody is prevented from purchasing several annuities. However, it remains unclear why under these circumstances differences in annuity premia seem to persist even among the ten largest insurance companies as shown by Friedman and Warshawsky (1988) and Mitchell, Poterba, and Warshawsky (1997). One possible explanation may be that current markets are very small and different insurance companies differ in overhead costs depending on how and to whom they sell their products.

6. Aside from this important assumption, the characteristics of insurance market equilibria have been found to be dependent on their specific assumptions about the order of 'moves' in the contracting game, as Hellwig (1987) points out.

7. Note that cooperative game theory provides another solution to the problem of unobservable annuity purchases. See Kahn and Mookherjee (1994).

3. A Two Period Model of Annuity Demand

This section develops the basic model used for the evaluation of adverse selection on annuity prices, annuity demand and welfare. The model is similar to Eckstein, Eichenbaum, and Peled (1985) but employs the type of equilibrium used by Abel (1986). The analysis is extended by allowing for heterogeneity in earnings and longevity.

3.1 The Basic Model

Consider an economy with N types of consumers, $j \in \{1,N\}$, who live for a maximum of two periods. Survival at the end of the first period of life is uncertain and occurs with probability π^j, $0 < \pi^j < 1$. Agents exogenously supply one unit of labor during their first period of life and receive labor income w^j. Survivors retire in the second period of their life. Note that each type j can hence be perfectly characterized by the pair $\{\pi^j, w^j\}$. Income and survival probabilities may be correlated.

Consumers choose consumption in both periods $\{c_1, c_2\}$ to maximize expected utility from a time-separable utility function. Future utility is discounted with the pure rate of time preference δ,

$$(1) \qquad \max \; U(c_1^j) + \frac{\pi^j}{1+\delta} \; U(c_2^j).$$

In order to smooth consumption over the expected lifespan agents can hold their wealth in form of bonds b^j that offer a rate of return r. The price of a bond is accordingly $1/R$ where $R=1+r$. Alternatively, individuals can buy an annuity a^j that is paid in year 2 conditional on the agent's survival. An annuity a^j can be purchased in period 1 for a premium Z^j. Since the insurance company implicitly distributes the estate of a deceased annuity purchaser among all surviving annuitants, the annuity can always offer a higher rate of return than a bond. Bonds are therefore a dominated asset and all agents in this economy would like to hold their wealth in annuities (Yaari,1965).[8]

Social security may affect the intertemporal allocation of consumption. Social security benefits are assumed to be paid according to current benefit rules, i.e. benefits do not generate the same rate of return for all members of a specific cohort. Instead, they depend on current law that generally redistributes within cohorts, from males to females and from singles to couples. Note also that social security does not coincide with an actuarially fair annuity based on average survival probabilities due to intergenerational redistribution.

A payroll tax with tax rate s finances social security benefits SS^j, which in turn are determined by a function of w^j (the social security benefit rule). Equations (2) and (3) represent the budget constraints for the consumer in both periods of his life.

$$(2) \quad c_1^j = w^j(1-s)-a^j Z^j$$

$$(3) \quad c_2^j = a^j + SS^j$$

Solving the intertemporal maximization problem as specified by the utility functions and the budget constraints yields an annuity demand function $a^j=a^j(Z^j, \pi^j, w^j, s, SS^j)$ for each type j. I make the following assumptions:

A1. *The utility function is twice continuously differentiable with $U'>0$ and $U''<0$.*

A2. *Annuity demand is non-negative, i.e. $a \geq 0$.*

8. This result depends on the assumption that agents do not have a bequest motive. Fischer (1973), Abel (1986) and Friedman and Warshawsky (1990) include a bequest motive in their respective models. Hurd (1989) does not find that a bequest motive is empirically relevant and asserts that people are in fact overannuitized by social annuities. Bernheim (1991), using the same data set as Hurd, finds that the data is consistent with bequest motives.

Assumption 1 is standard and ensures that annuity insurance demand is continuous in Z.[9] Assumption 2 warrants some discussion. If the government provides social security benefits some agents may be overannuitized and would prefer to borrow against future annuity income by selling annuities. Equivalently, as shown by Yaari (1965), they may borrow and simultaneously buy life insurance. The life insurance guarantees the loan in case of death. Assumption 2 therefore rules out the possibility that individuals can draw loans on future social annuity income and guarantee repayments through life insurance.[10] Note also, that the condition is never binding in a two period model unless the government provides social security since all agents want to be able to consume during the second period of their live should they survive.

Given the first two assumptions, annuity demand has to satisfy the following first order condition:

$$(4) \quad U^{\prime}(c_1^j)\ Z^j \ \geq \ \frac{\pi^j}{1+\delta}\ U^{\prime}(c_2^j)$$

This equation also represents the conventional Euler equation for consumption with the premium Z^j replacing the discount factor *1/(1+r)*. Due to assumption 2, equation (4) may hold with inequality if agents are overannuitized by social security.

Everything else being equal, annuity demand needs to increase with survival probabilities in order to satisfy the first order condition if the latter holds with equality. Hence, higher survival probabilities induce higher annuity demand. Moreover, higher income implies higher consumption in both periods and thus higher annuity demand as long as social security benefits are not regressive. Finally, if part of second period consumption is provided by social security benefits, annuity demand must fall to satisfy equation (4).

Besides A1 and A2, I make the following assumptions:

9. This is a standard textbook result of consumption theory.

10. Friedman and Warshawsky (1988,1990) make the same assumption in their simulations. Abel (1986) restricts his parameter space such that annuity demand is non-negative.

A3. *Each type j comprises a share* ϵ^j *of the population with* $\sum_j \epsilon^j = 1$.

A4. *The rate of return r of riskless assets is fixed.*

A5. *Insurance companies have no information about the type of an annuity purchaser. They cannot efficiently monitor the overall amount of annuities purchased by their customers and can therefore not restrict the quantity purchased. Insurance companies can observe age and sex but not income.*

The fourth assumption excludes general equilibrium feedback effects on interest rates.

Assumption 5 generates the adverse selection problem by eliminating the first best outcome in which annuities can be purchased based on individual survival prospects. It also eliminates the Rothschild-Stiglitz and Wilson separating and pooling equilibria since the latter rely on the assumption that the quantity of insurance can be constrained.

Insurance companies set the premium for insurance only, they cannot control the quantity. Hence, the type superscript is dropped from the annuity premium from now on. By furthermore assuming that income is non observable, I also exclude the possibility that income is used as a monitoring device. This is realistic if annuity insurance companies do not handle payroll or total assets for the annuity purchaser and cannot collect information about his wealth and income. Since an annuity purchaser can split his contract into several pieces, he can then effectively hide his financial status. Given the positive correlation between income and mortality types with high income would like to pretend to be less wealthy to reduce their annuity premia.

According to assumption 5 insurance companies charge a premium Z per dollar of annuity payments in period 2, irrespective of the type. An annuity a^j sold to a type j customer will generate expected profits equivalent to the premium charged net of the discounted value of the expected future annuity payment. Therefore, expected profits P for the type j annuity contract are equal to $(Z-\pi^j/R)\, a^j$. In a competitive insurance market, the premium will be sufficient to break even across all types of customers in equilibrium. Accordingly, Z is implicitly defined by the zero profit condition,

$$(5) \qquad P(Z) = \sum_{j=1}^{N} \epsilon^j \; (Z-\pi^j/R) \;\; a^j(Z,\pi^j,w^j,s,SS^j) = 0.$$

By extending Abel's (1986) argument, it can be shown that there exists at least one annuity premium that satisfies equation (5). The formal arguments are provided in Appendix 1. If social security benefits are small enough, there always exists a zero profit equilibrium for those agents participating in the annuities market. If social security benefits are sufficiently large and all agents are overannuitized, however, the only zero profit equilibrium for the annuities market may be one in which nobody demands private annuities.

The equilibrium may, in general, not be unique and depends on the specifics of the utility function and the according behavior of annuity demand functions. Nevertheless, for a given specification, the uniqueness can be tested by deriving the properties of $P(Z)$. If $P(Z)$ is strictly concave on the range $(0,Z^{max})$ the equilibrium will be unique.

As Abel (1986) has shown, equilibrium annuity prices will be higher than a premium based on population average survival. This result is obviously related to adverse selection: Consumers with higher than average survival prospects purchase larger annuities than those with below average survival probabilities. Consequently, prices based on population average mortality cannot sustain a zero profit equilibrium.

In the extended model presented here it can be shown that the result holds as long as income and longevity are not negatively correlated and social security benefits are not regressive (see Appendix 2). Empirically, both assumptions hold. The correlation between income and longevity then exacerbates the problem of adverse selection.[11] Individuals with higher income will demand more annuities for two reasons, their higher chances to survive and their higher income. In other words, people who live longer on average will occupy a larger share of the annuities market if longevity increases with income which drives annuity premia up.

This result raises the important issue how to measure adverse selection in the presence of a positive correlation between income and longevity. The traditional approach followed by Friedman

11. For results from the PSID and an overview of the literature see Lillard and Waite (1995) and Lillard and Panis (1996). Similar findings for the National Longitudinal Survey have been reported by Menchik (1993) who concludes that "differential mortality by economic status is strongly present in the United States". For evidence from the SIPP see Attanasio and Hoynes (1995). A broad survey of the literature can be found in Feinstein (1993). Medical research by Pappas et al. (1993) also concluded that the disparity in mortality between socioeconomic groups is significant and has increased over time.

and Warshawsky (1988) is to calculate a "load factor" that measures by how many percent the asking price Z^* of an annuity insurance company exceeds the actuarially fair price. The latter is calculated simply as the price based on the average mortality of the population. For the two period model the load factor can be derived as

$$(6) \qquad \frac{Z^*}{Z^F} = \frac{Z^* \; R}{\sum_{j=1}^{N} \epsilon^j \pi^j} = \frac{Z^* \; R}{\bar{\pi}}$$

where Z^F denotes the premium of an annuity based on average mortality.

Some portion of the load factor derived in equation (6) is due to the positive correlation between income and longevity. If all agents were identical in their survival probabilities and agents were pooled for an annuity, type *j* 's annuity would differ from type *k*'s by the ratio of their lifetime income in present values. Therefore, one way to reflect the effect of the correlation between income and mortality is to calculate the price of an annuity based on a lifetime income weighted mortality. This "corrected" load factor can be derived as follows:

$$(7) \qquad \frac{Z^*}{Z^S} = \frac{Z^* \; R \sum_{j=1}^{N} \epsilon^j w^j}{\sum_{j=1}^{N} \epsilon^j \pi^j w^j}$$

Both measures together can provide information about the size and sources of the measured actuarial unfairness of annuities markets. If the loading according to equation (7) is large and close to the conventional load factor, then a mandated annuity rising with income must be more expensive than average mortality predicts.

3.2. Analytical Results for the Log Utility Case

To solve analytically for the annuity premium in a private annuity market, I assume in the following that utility is logarithmic. Accordingly, the lifetime utility function can be expressed as:

$$(8) \qquad U = \log(c_1^j) + \frac{\pi^j}{(1+\delta)} \log(c_2^j).$$

Annuity demand can then be derived as follows from the first order condition and the budget constraints provided above:

$$(9) \qquad a^j = \frac{w^j \pi^j (1-s) - Z(1+\delta) SS^j}{(1+\pi^j+\delta)\ Z}$$

as long as this value exceeds zero.

As discussed in the previous section, annuity demand increases with wage income and the survival probability and decreases with the price of annuities. Lifetime resources are divided between the two periods of life according to the personal discount factor. The latter depends on the type-independent pure rate of time preference and the type-specific survival probability. With increasing survival probabilities, agents discount future consumption less and annuity demand increases.

Equation (9) shows clearly that annuity demand decreases with the size of social security. The larger the premium for private annuities the more important is the annuity provided by the government. In fact, the relative marginal change of annuity demand $\frac{\partial a^j / \partial SS^j}{a^j}$ decreases with higher survival probability as long as $\frac{\partial w^j}{\partial \pi^j} \geq 0$ and $\frac{\partial^2 SS^j}{\partial (w^j)^2} \leq 0$. The latter holds if income and longevity are positively correlated and the social security system is progressive as in the United States.

As a consequence of the stronger decline in annuity demand for individuals with smaller income and shorter life expectancy, prices for private annuities will have to rise in equilibrium in presence of social security. Social security replaces a larger percentage of annuity demand for those who represent very favorable risks for insurance companies which in turn forces premia to rise in response to social security (see also Abel, 1986). Currently observable annuity prices hence inseparably reflect both the presence of social security benefits and the effect of adverse selection.

As an example for the calculation of load factors, consider next how the load factor for annuity insurance turns out for Cobb-Douglas utility. Equation (10) displays the formula for a case without social security, and Appendix 3 solves for the annuity insurance load factor if social security benefits are available.

$$(10) \qquad \frac{Z^*}{Z^F} = \frac{1}{\bar{\pi}} \frac{\sum_{j=1}^{N} \epsilon^j \pi^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} \right)}{\sum_{j=1}^{N} \epsilon^j w^j \left(\frac{\pi^j}{1+\pi^j+\delta} \right)}$$

I assume that 9 distinct types with survival probabilities ranging from 0.1 to 0.9 represent the probability distribution.

Solving (10) for a case with approximately Normal(0.5,0.2) distributed survival probabilities, zero correlation between income and mortality, and pure rate of time preference $\delta = 0.35$, generates a load factor of 12.3 percent. In other words, equilibrium annuity prices would exceed prices based on average mortality rates by 12.3 percent. Adding positive correlation of 0.5 between income and mortality almost doubles the standard load factor to 24.4 percent. The load factor that corrects for the correlation between income and mortality indicates only 7.4 percent higher prices. Therefore, the correlation between income and longevity accounts for about 17 percent of the observed "unfairness" of annuity prices. Furthermore, introducing intragenerationally fair social security benefits financed by 10 percent of wages raises the load factor to 20.9 percent if income and mortality are not correlated.

Those results clearly show that a model of annuity prices needs to incorporate both the correlation between income and longevity and the generosity of social security benefits. The example also makes clear that a conventionally measured load factor may disguise the importance of the effect of the correlation between income and mortality.

4. Adverse Selection and Annuity Insurance -- How Well Does the Life Cycle Model Match the Data?

4.1 Extending the Model to 75 Years

The model of annuity demand can now be extended to 75 years (ages 25 to 100). A type *j* consumer maximizes the following utility function:

$$(11) \qquad \sum_{t=1}^{75} U(c_t^j) \prod_{s=1}^{t-1} \frac{\pi_s^j}{(1+\delta)}$$

where π_t^j stand for the probability that type j lives to from age t to age $t+1$. The maximization is subject to a sequence of budget constraints evolving as follows:

$$(12) \qquad c_t^j = w_t^j(1-s) + a_t^j - a_{t+1}^j Z_t^* + SS_t^j$$

with social security benefits set to zero before retirement and wages set to zero after retirement. Annuity demand is zero in the last period of life. Note also that the weights of the different types now change over time according to the following equation:

$$(13) \qquad \epsilon_t^j = \epsilon_{t-1}^j \frac{\pi_{t-1}^j}{\bar{\pi}_{t-1}}.$$

In extending the two period model to 75 years of life, an additional assumption is required:

A.6 *Survival probabilities are strictly ranked across types.*

Assumption 6 ensures that a type *j* with higher chances to live from age 50 to age 51 than type *k,* say, also has a higher probability to live from age 51 to 52 than type *k* and so on. This assumption is satisfied if socioeconomic characteristics shift survival probabilities proportionally, an assumption used for example by Lillard and Waite (1995). This assumption ensures that individuals cannot improve their welfare by designing a multi-period contract among a subgroup of types. Effectively, the type with the highest survival probability has the highest survival probability throughout all periods of life and would therefore always prefer to disguise his longevity characteristics in order to reduce the cost of his annuity. The same holds for the type with the next highest survival probability and so on.

Under assumptions A1 to A6 the multi-period equilibrium for annuities can be defined as a sequence of single period contracts satisfying equation (5). The existence of this multi-period equilibrium can be inferred from the existence of zero profit premia for each period.

The price of an annuity spanning several time periods can then be calculated from the sequence of prices prevailing in each period. In particular, the premium PR_t of a \$1 life annuity in time t generated by the model equals:

$$(14) \qquad PR_t = Z_t^* + Z_t^* Z_{t+1}^* + Z_t^* Z_{t+1}^* Z_{t+2}^* + \ldots$$

Accordingly, the actuarial fair premium PR_t^F can be calculated as:

$$(15) \qquad PR_t^F = Z_t^F + Z_t^F Z_{t+1}^F + Z_t^F Z_{t+1}^F Z_{t+2}^F + \ldots$$

and the percentage deviation of annuity prices from actuarial fairness ("load factor") is just the ratio of PR_t and PR_t^F . A similar derivation holds for the annuity price that uses an income weighted average population survival probability.

4.2 Calibrating the Model

To reproduce an observable degree of adverse selection in the annuities market, the calibration has to reflect the empirical distribution of survival probabilities. Vital statistics can only provide limited information on the full spread of survival probabilities. They also do not reflect the downward trend in mortality rates.

This paper therefore takes a different approach for the calibration than other models with longevity uncertainty: Empirical estimates from Lillard and Panis (1996) are applied to an entire cohort of individuals from the Current Population Survey. In order to reproduce the empirically meaningful distribution of income and marital status the calibration starts with the cohort of 50-year-old individuals in the 1995 CPS, a total of 1458 individuals, 721 men and 737 women. I exclude about 180 individuals with either family income below \$5,000 or without labor income and a working spouse. Age 50 is convenient since there are few future changes in marital status outside of changes induced by the death of a spouse. On the other hand, early retirement does not yet affect income measures. Moreover, before age 50 mortality probabilities are comparatively small. Population weights for each of the 50-year-old individuals are also obtained from the CPS.

Lillard and Panis (1996) use data from the PSID to estimate parameters for a time dependent baseline mortality hazard function. Marital status, race, educational attainment and lifetime family income then shifted this function proportionally. Lillard and Panis find a strong positive effect of lifetime family income on survival probabilities. Accounting for this effect wipes out the effect of current income identified by Lillard and Waite (1995). Figure 1 illustrates the effect of income on mortality rates for a married male with $20,000 and $180,000 family income at age 50, respectively.[12]

Table 1 summarizes the relevant characteristics of the cohort used for the calibration. The largest variation is in family income which has a mean of $65,000 for males and $62,000 for females. The standard deviation of family income is quite large and the variation in family income thus contributes greatly to the variation in survival probabilities. To apply the parameters estimated by Lillard and Panis, family income from the CPS is adjusted proportionally such that mean family income coincides with mean permanent income in the PSID sample.

Additional sources of variation in longevity are educational attainment, marital status, and to a lesser extent race. After including permanent income Lillard and Panis find no significant effect of race on the mortality rate of males. For married agents characteristics of the spouse play a role in survival probabilities since the potential death of the spouse has negative effects on own survival prospects. I include this effect into the survival probabilities after age 50.[13] For years of life between age 25 and age 50 the survival probabilities are assigned according to the characteristics of the person at age 50.

The calibration captures potentially observable factors of heterogeneity that, however, have

12. Note that due to the lack of income shocks in the model family income is closely related to lifetime family income.

13. See also Lillard, Brien, and Panis (1996) for a derivation of survival probabilities when future events are unknown.

so far not been used by the insurance industry to predict survival probabilities. Due to regulation and difficulties in monitoring characteristics such as family income, these factors probably never will play a role in calculating prices for private annuities. The model does not capture other potentially unobserved factors determining mortality.

Applying the empirical parameters to the CPS cohort generates an average death probability of 0.0065 for males and 0.0035 for females. These figures are very close to the corresponding values of 0.0062 and 0.0035 in the 1991 life tables. The survival probabilities are nevertheless readjusted to coincide perfectly with averages in the life tables.

Due to the time dependence built into Lillard and Panis' estimation, survival probabilities for ages higher than 50 are below those of current vital statistics. In fact, the probabilities derived for the CPS cohort imply a further increase in life expectancy of around 3 years for males and 4 years for females. The future decline in mortality is an important factor in the determination of annuity demand (Friedman and Warshawsky, 1990).

In order to specify income paths for agents I assume that agents retire at age 65 and that -- taking labor income at age 50 as a benchmark -- wage profiles have the same curvature as the mean efficiency profile estimated by Altig et al. (1997). Since labor supply is exogenous, efficiency profiles are additionally adjusted for the change in hours worked to generate earnings profiles.[14] Earnings are furthermore corrected for income tax payments. For this purpose, I apply the statutory tax code, taking into account marital status and estimated changes in itemized deductions with income (see Altig et al.,1997). [15]

For couples labor income of husband and wife are added and divided by two. Since labor supply is exogenous, this implies that each spouse's annuity demand is based on half of the joint resources. This approach approximates the spousal benefit rules and ownership rules of retirement

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14. See Rupert, Rogerson, and Wright (1996) for an estimation of average changes in hours worked from the Michigan Time Use Survey.

15. Note that the assumption of exogenous retirement behavior excludes a possible interaction between retirement choices and the cost of private annuities (see Rust and Phelan, 1997). However, by benchmarking income against the CPS, some of the endogenous labor supply choices are implicitly incorporated into the calibration and as long as the characterization of lifetime incomes is not biased to a major extent, the exogenous labor supply should not have a large effect on the results.

accounts in a private retirement system.[16]

Social security benefits are calculated according to the current benefits rules and take spousal benefit rules as well as the payroll tax ceiling into account. In line with the division of resources mentioned above, each spouse is assigned half of the joint social security benefits.

All numerical simulations assume that utility is additively separable and takes the standard constant relative risk aversion form $U(c_t) = (1/1-\gamma)\, c_t^{1-\gamma}$. The exponent γ simultaneously determines the risk aversion of economic agents and their willingness to intertemporally substitute consumption. The higher γ the flatter is the desired consumption path and the higher is the degree of risk aversion. It is generally believed that reasonable values for the degree of risk aversion range from 2 to 4. I will consider below how results change with the risk aversion parameter ranging from 1 (log utility) to 4. The pure rate of time preference is set to 1 percent or 3 percent, and the fixed real rate of return on bonds r is set to 3 percent in all subsequent runs.

4.3. Solving the Model

The 75-period-model is solved with an iterative procedure. First, the optimal forward-looking consumption path for a given set of annuity prices is calculated for each type. Second, the annuity demand is used to derive expected profits according to equation (5). Third, for given annuity demand zero profit annuity prices are calculated and a weighted average of the old price and the new price is assigned as the annuity premium for the next iteration. This procedure is repeated until profits reach zero, which usually takes not more than 20 to 30 iterations.

Due to the non-negativity constraint computations can get more burdensome, though, if social security is switched on. Here, solving the optimal forward-looking consumption path itself necessitates an iterative procedure. If the non-negativity constraint is violated, the algorithm assigns a positive value to a Kuhn-Tucker multiplier capturing the difference in marginal utility between the

16. Note that the sharing rule differs from joint maximization of annuity demand since couples may also share longevity risk within the family (Kotlikoff and Spivak, 1981). However, the focus of this paper is on single life annuities.

two periods, and the consumption path is subsequently recomputed.[17] This procedure is repeated until the violation of the non-negativity constraint falls below a certain threshold. The consumption paths obtained by these calculations then determines the annuity demand and expected profits for annuity insurance companies.

As mentioned earlier, the zero-profit equilibrium may not be unique for all parameter choices. This result is true even if utility is restricted to the constant relative risk aversion type. Only if utility is logarithmic and social security is unavailable the profit function is strictly concave. In all other cases the equilibrium properties depend on the particular parameter choices. The results presented below are invariant to the starting value of the simulation. However, that does not necessarily rule out other equilibria.

4.4. Adverse Selection in Annuities Markets -- What a Life Cycle Model Predicts

The Current System

If economic agents behave according to the life cycle model, to what extent would annuity prices differ from prices based on average survival probabilities? Table 2 exhibits the load factors generated by the calibrated life cycle model for ages 55, 65, and 75 and different risk aversion parameters of the utility function. The runs assume a pure rate of time preference of 1 percent. Table 3 presents the runs for a pure rate of time preference of 3 percent.

If consumers receive social security benefits, the model predicts that adverse selection will raise annuity prices for 65-year-old males by 6.5 percent to 10.5 percent above those based on the average mortality of the population. These load factors are similar to those calculated by Mitchell, Poterba and Warshawsky (1997), summarized in Table 4. They find values for 65-year-old males between 8 and 10 percent, depending on the discount rate. However, the sources of adverse selection partly differ between the theoretical model and the empirical calculations, since the load factor in the model also reflects the heterogeneity of annuity purchasers. Mitchell, Poterba, and Warshawsky instead exclusively focus on the difference between annuity purchasers and the average population.

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17. Zeldes (1989) derives the first-order conditions of consumer choice with non-negativity constraints.

The deviation from prices that are actuarially fair due to adverse selection depends on the degree of risk aversion. If economic agents are not very averse to risk -- as in the log utility case -- demand is more responsive to the difference between the individual discount factor and the market rate of return. The first figure combines the pure rate of time preference and individual survival probabilities, and the second figure reflects the price of annuities (the inverse of the rate of return). The higher is the degree of risk aversion the less differs annuity demand among agents and the smaller is the adverse selection problem. Reasonable risk aversion parameters range between 2 and 4. In addition, the effect of social security benefits on annuity demand and adverse selection depends on the pure rate of time preference. The difference between the "With Social Security" columns in Tables 2 and 3 reflects this result.

A second result of the model is that load factors vary with age. According to the second row of Table 3, under the current system an annuity purchased by a male at age 55 is predicted to be 6.5 percent more expensive than a fair annuity. The loading rises to 8.5 percent at age 65 and 10.2 percent at age 75. However, between age 85 and 90 the load factor starts to fall. The dependency of adverse selection on age is a consequence of the endogeneity of the population weights. The higher death rates of some types cause a fall in their population shares. Thus, the types with longer life expectancy exert a larger influence on the market without causing the average life expectancy to rise to the same degree initially. In later years of life, there are almost no types with short life expectancy left. Therefore, the average agent is a long lived agent and load factors fall.

Mitchell, Poterba and Warshawsky (1997) also find a significant increase in adverse selection induced loading of annuity prices with age. Depending on their discount rate, prices are 5.1 to 6.6 percent higher for 55 year old males and between 10.8 and 12.5 percent higher for 75-year-old males. Whereas the model matches the stylized fact of increasing load factors with age it predicts a smaller increase between ages 55 and 75.

The results from the model as displayed in Figure 3 are also in line with the empirical finding that adverse selection affects annuity prices for females to a lesser extent. The life cycle model exhibits load factors for women that are about 20 percent smaller than those for males. Mitchell, Poterba, and Warshawsky on the other hand find that the loading of annuities sold to women is around 35 percent smaller.

Lower values for women can largely be attributed to lower variances in the factors determining survival. First, the variance of family income is smaller for women than for men. The variance in educational attainment is smaller as well with higher concentration at high school degrees. Second, survival probabilities for women as estimated by Lillard and Panis (1996) vary to a much lesser degree with marital status than those for men. In fact, the women who are never married or widowed are not facing a significantly different life expectancy than married women. Moreover, divorced and separated women live slightly longer than married women. Overall, the variance for female survival probabilities is therefore considerably smaller than the variance in survival probabilities for men. This difference explains the lower degree of adverse selection in the market for females.

The Impact of Privatization

To what extent does the existence of social security matter? According to the model, eliminating social security benefits may lead to a drop in the load factor by up to 2.7 percentage points. As shown by Figure 3, the drop occurs at all ages. Depending on the degree of risk aversion, the reduction of adverse selection implies a 12 to 30 percent fall of existing load factors. Generally speaking, the reduction is stronger for higher degrees of risk aversion and the higher rate of time preference. Note, though, that the ultimate load factor in a system without social security is independent of the rate of time preference δ. Therefore, columns 2 and 4 in Table 2 and 3 are identical. This result follows because load factors represent ratios of weighted annuity demand and the pure rate of time preference affects numerator and denominator in an identical way. Equation (10) can be employed to derive this result formally.

Results for the percent reduction in load factors are similar for males and females. This finding follows from the increasing annuity demand of those with below average survival probabilities. Indeed, the increasing demand of those with shorter lives has two driving forces: as shown by Abel (1986) privatizing social security increases the demand of those with below average survival probabilities more than proportionally. This result alone reduces the adverse selection problem. However, a second feature of the model exacerbates this effect. Social security benefits are progressive, and longevity and income are positively correlated. Therefore, social security benefits replace a much larger proportion of wealth for the poor who also have shorter lives expectancies. If those types enter the private market, the prices for annuities will fall in response.

Tables 5 shows the change in annuity demand and the share of social security wealth for married agents aged 50. The selected incomes are $10,000, $30,000, $50,000 and $90000, respectively.[18] Figures 4 and 5 display consumption and asset paths for a married man with $10,000 and $90,000 income, respectively, for a utility function with $\gamma = 2$. The table and figures reveal the source of the change in annuity demand after privatization. Richer agents hold a much smaller share of their wealth in social security wealth. In fact, since the model is calibrated to current benefit rules, it matches empirical findings about social security wealth by Gustman et al. (1997) fairly well. Social security wealth in the model, discounted with the rate of return on annuities, ranges from above 80 percent of total wealth for the poorest to below 20 percent for high earners. Average earners hold about 50 percent of their wealth in social security. Because removing social security benefits increases demand for annuities by consumers with below average survival prospects more than proportionally, prices for annuities and measured adverse selection fall.

Note also that the difference in mortality rates and the resulting difference in individual discount factors results in a large possible variety of consumption paths. An agent whose survival probabilities drop below the discount factor implied by annuity prices exhibits falling consumption. It is therefore likely that poorer agents prefer to consume more resources earlier than the current social security system allows. Figure 4 displays an outcome where social security constrains agents to consume more than is optimal after age 85.

An important feature of the model is that it captures the positive correlation between lifetime income and longevity. As outlined in the previous section, any empirically measured loading of annuity prices implicitly incorporates the effect of this correlation on annuity demand. In order to isolate the impact of the income-longevity correlation, Tables 6 and 7 display the "corrected" load factor. It compares equilibrium annuity prices with prices based on lifetime-income-weighted survival probabilities. The price calculated with the lifetime-income-weighted mortality measure,

18. Recall that income allocated to spouses reflects half of the joint income of a couple. $30,000 reflects the mean sample income per individual after making this adjustment.

in turn, reflects the cost of a mandatory annuity that rises proportionally with income. Consequently, the "Without Social Security" columns of Table 6 and 7 show that a mandatory annuity that rises proportionally with income and is financed with proportional payroll taxes could only reduce the observed loading by 60 to 80 percent for a 65-year-old male. The lower is the degree of risk aversion the more important is the positive correlation between income and longevity in explaining the loading of annuity prices. Therefore, a mandate providing an annuity that rises with income is less powerful for lower degrees of risk aversion.[19] As a result, an important question for the regulation of a privatized retirement system is how to design the mandate, i.e. whether and how to reduce the relationship between the size of the annuity and lifetime income to reduce the cost of annuities. This question is explored in more depth in Walliser (1997).

Annuity Prices and Welfare

A remaining question that needs to be addressed is the change in lifetime utility under both regimes, with and without social security, as compared to the first best outcome. As pointed out earlier, an omniscient planner could use all the information related to mortality and potentially predict survival probabilities based on several characteristics. Those survival probabilities could then be used to separate the market and assign annuity prices based on individual survival probabilities rather than pooling agents. Table 8 provides an overview to which extent lifetime utility, measured in wealth equivalents, differs from this first best outcome if agents are pooled. Given the higher probability of death for agents with lower income, it is hardly surprising that the latter agents suffer in a pooled market. Pooling thus implicitly redistributes from agents with below average income to those with above average income. Depending on the pure rate of time preference, the utility loss for males ranges from 2.6 percent to 3.7 percent of lifetime income if social security is unavailable. Similar values hold for married females.

The redistribution is substantially mitigated if the government provides social security benefits, since benefits are progressive. However, the implicit redistribution through pooling is not

19. The importance of the positive income mortality correlation is smaller for women with load factors in Table 5 and 6 that are 80 to 90 percent lower than those in Tables 2 and 3.

eliminated. Note also that the married males belong to the winners of pooling at lower income levels than females. This result reflects the empirically important correlation between marital status and longevity for men. Thus, married men's annuity prices fall when they are pooled with unmarried men. A similar relationship does not hold for women, which is why married women with mean income still belong to the losers of pooling.

Since the columns in Table 8 display the percentage change in utility for different annuity prices keeping the respective system in place, the results do not indicate whether abandoning social security will actually lead to utility gains or losses. Clearly, if current social security benefit rules prevail, the implicit rate of return on social security benefits will be lower for most individuals than the assumed 3 percent market rate of return in the calculations presented above. In that case, removing the progressivity of the social security system may be more than outweighed by the increased return and reduced forced annuitization. However, for such an analysis to be complete, one would need to address the transition issue as well as possible general equilibrium effects as in Kotlikoff, Smetters, and Walliser (1997).

Limitations of the Model and Future Research

As explained previously, the model captures a number of empirical regularities of annuity prices. However, it still predicts higher annuity demand than empirically is observed. At minimum, 50 percent of the agents in the model purchase private annuities at any given age. In contrast, empirically fewer than 5 percent of the population purchase private annuities. As a consequence, the model attributes a portion of the measured adverse selection to the heterogeneity within the pool of annuitants rather than deriving adverse selection from the difference between annuity purchasers and non-purchasers.

A whole range of factors may explain why annuity demand is higher in the model. First, the model does not incorporate a bequest motive. Including a bequest motive would lead more agents to hold non-annuitized wealth. Nevertheless, even with a bequest motive the mortality distribution of annuitants may not change substantially if the desire to bequeath increases with income. The exact effect remains to be seen from a future extension of the model.

Another factor that may contribute to higher than observable annuity demand in the model is the lack of private pensions and other annuitized resources. According to Gustman et al. (1997) agents with above average earnings may hold up to 30 percent of their wealth in private pensions. Furthermore, other annuitized government programs have grown over time, specifically Medicare (Gokhale, Kotlikoff, and Sabelhaus, 1996). A high share of annuitized wealth may be responsible for the lack of private annuity demand. Moreover, a large portion of assets for lower income households is also locked in housing equity. If the market for reverse mortgages is underdeveloped it may be difficult to annuitize housing wealth. Also, as Kotlikoff and Spivak (1981) show, families may self-insure against longevity risk.

Finally, overhead costs of annuity insurance providers may lead to actuarial domination of annuities, as shown by Friedman and Warshawsky (1990). Also, if overhead costs vary with the size of the annuity, the fact that richer agents buy larger annuities may lead to economies of scale that partially offset the lower mortality of richer agents. Some anecdotal evidence seems to point in this direction.

5. Conclusion

A life cycle model that explicitly captures the heterogeneity of survival prospects of a single cohort can account for three stylized facts: The size of adverse selection, the increasing importance of adverse selection with age, and the smaller adverse selection in the annuities market for women. The model also shows that eliminating social security benefits can decrease the load factor by 2 to 3 percentage points, a reduction of 20 to 30 percent. The reduction in adverse selection is driven by the strong increase in annuity demand of shorter lived poor agents who currently receive the most generous returns on their social security contributions. However, privatization cannot remove a significant portion of the adverse selection in the annuities market.

The progressivity of social security benefits mitigates the implicit redistribution involved in pooling if income and longevity are positively correlated. A privatized system that does not provide benefits on a progressive basis therefore increases the redistribution from the poor to the rich. However, as we show elsewhere, a possibly higher rate of return on private retirement savings combined with favorable general equilibrium effects may lead to long run gains in a privatized system even for low income individuals despite the removal of a progressive system (Kotlikoff, Smetters, and Walliser, 1997).

A significant part of the measured adverse selection can be attributed to the correlation between income and longevity. Reducing adverse selection by mandating annuitization in a privatized system must take this relationship into account, a point explored in more detail in Walliser (1997).

Future research will account for more factors that may explain the small size of observed private annuity markets. Possible extensions include a bequest motive, which depends on lifetime income, pension wealth, and transaction costs.

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Table 1: Sample Statistics for 1995 CPS Cohort of Age 50 (weighted, in percent)

	Males	Female Spouses	Females	Male Spouses
Mean Family Income (1995 Dollars)	65212	70487	62078	70416
Standard Deviation of Family Income (1995 Dollars)	18443	18346	18040	18110
Less Than High School	16.0	10.0	11.8	12.8
High School	27.9	38.6	33.9	27.8
More Than High School	56.1	51.2	54.4	59.4
Non-White	10.5	10.0	15.1	10.7
Married	82.2	100.0	75.6	100.0
Never Married	4.8	0.0	4.5	0.0
Divorced	10.2	0.0	13.9	0.0
Separated	2.4	0.0	2.8	0.0
Widowed	0.5	0.0	3.3	0.0
Age (in years)	50.0	46.8	50.0	52.6

Table 2: Difference between the Equilibrium Price of a Perpetual Annuity and the Price of an Annuity Based on Average Survival Probabilities, **r=.03, δ=.01 (in percent)**

		Men		Women	
Rate of Risk Aversion	Age at Purchase	With Social Security	Without Social Security	With Social Security	Without Social Security
$\gamma = 1$	55	6.3	5.3	4.4	3.7
	65	8.3	6.9	5.9	5.0
	75	10.0	8.4	7.5	6.4
$\gamma = 2$	55	5.6	4.4	4.2	3.4
	65	7.2	5.7	5.6	4.4
	75	8.6	6.7	6.9	5.5
$\gamma = 4$	55	5.1	3.9	4.1	3.1
	65	6.5	4.9	5.4	4.1
	75	7.6	5.7	6.6	5.0

Table 3: Difference between the Equilibrium Price of a Perpetual Annuity and the Price of an Annuity Based on Average Survival Probabilities, r=.03, δ=.03 (in percent)

		Men		Women	
Rate of Risk Aversion	Age at Purchase	With Social Security	Without Social Security	With Social Security	Without Social Security
$\gamma = 1$	55	7.8	5.3	5.6	3.8
	65	10.3	7.0	7.7	5.0
	75	12.5	8.5	9.9	6.5
$\gamma = 2$	55	6.5	4.4	4.9	3.4
	65	8.5	5.7	6.6	4.4
	75	10.2	6.8	8.3	5.5
$\gamma = 4$	55	5.5	3.9	4.5	3.1
	65	7.1	4.9	5.9	4.1
	75	8.4	5.7	7.3	5.0

Table 4: Adverse Selection Induced Increase in 1995 Annuity Prices (in percent)

	Men		Women	
Age at Purchase	Discounted With Treasury Bond Rate	Discounted With Corporate Bond Rate	Discounted With Treasury Bond Rate	Discounted With Corporate Bond Rate
55	6.6	5.1	4.6	3.5
65	10.0	8.3	6.4	5.2
75	12.5	10.8	6.9	5.9

Source: Mitchell, Poterba and Warshawsky (1997), Tables 3 and 4.

Table 5: Change in Annuity Demand for Married Agents at Age 65 and Share of Resources Provided by Social Security at Age 65, γ=2, r=.03 (in percent)

		Men		Women	
Income	Rate of Time Preference	Percentage Change in Annuity Demand	Share of Wealth in Social Security	Percentage Change in Annuity Demand	Share of Wealth in Social Security
$10,000	0.01	196.6	82.3	131.8	73.7
	0.03	665.7	94.1	292.5	86.5
$30,000	0.01	69.8	58.8	76.9	60.3
	0.03	111.5	70.4	130.2	72.3
$50,000	0.01	48.1	46.5	48.3	45.1
	0.03	68.4	56.8	69.3	55.4
$90,000	0.01	24.3	30.4	31.3	37.1
	0.03	33.7	38.2	44.0	46.3

Table 6: Difference between the Equilibrium Price of a Perpetual Annuity and the Price of an Annuity Based on Lifetime Income Weighted Average Survival Probabilities, r=.03, δ=.01 (in percent)

		Men		Women	
Rate of Risk Aversion	Age at Purchase	With Social Security	Without Social Security	With Social Security	Without Social Security
$\gamma = 1$	55	2.9	2.0	1.5	0.8
	65	4.1	2.8	2.1	1.3
	75	5.3	3.9	2.9	1.9
$\gamma = 2$	55	2.3	1.1	1.3	0.5
	65	3.1	1.6	1.8	0.7
	75	4.0	2.2	2.4	1.0
$\gamma = 4$	55	1.7	0.6	1.2	0.2
	65	2.4	0.9	1.6	0.4
	75	3.0	1.2	2.1	0.5

Table 7: Difference between the Equilibrium Price of a Perpetual Annuity and the Price of an Annuity Based on Lifetime Income Weighted Average Survival Probabilities, r=.03, δ=.03 (in percent)

		Men		Women	
Rate of Risk Aversion	Age at Purchase	With Social Security	Without Social Security	With Social Security	Without Social Security
$\gamma = 1$	55	4.4	2.0	2.7	0.9
	65	6.1	2.9	3.8	1.3
	75	7.8	3.9	5.2	2.0
$\gamma = 2$	55	3.1	1.1	2.0	0.5
	65	4.3	1.6	2.8	0.7
	75	5.5	2.3	3.7	1.1
$\gamma = 4$	55	2.2	0.6	1.6	0.2
	65	3.0	0.9	2.1	0.4
	75	3.8	1.2	2.8	0.5

Table 8: Change in Lifetime Utility (Wealth Equivalents) for Selected Married Agents, γ=2, r=.03, Compared to First Best (in percent)

		Men		Women	
Income	Rate of Time Preference	Compared to Actuarially Fair Price for Individual, With Social Security	Compared to Actuarially Fair Price for Individual, Without Social Security	Compared to Actuarially Fair Price for Individual, With Social Security	Compared to Actuarially Fair Price for Individual, Without Social Security
$10,000	0.01	-1.7	-3.7	-2.1	-3.8
	0.03	-0.1	-2.6	-0.9	-2.7
$30,000	0.01	1.0	2.0	-0.7	-0.8
	0.03	0.4	1.4	-0.4	-0.5
$50,000	0.01	1.4	2.3	1.2	1.9
	0.03	0.7	1.6	0.6	1.4
$90,000	0.01	2.7	3.6	2.2	3.2
	0.03	1.6	2.5	1.3	2.2

Figure 1: Death Probabilities for Selected Married Male Agents

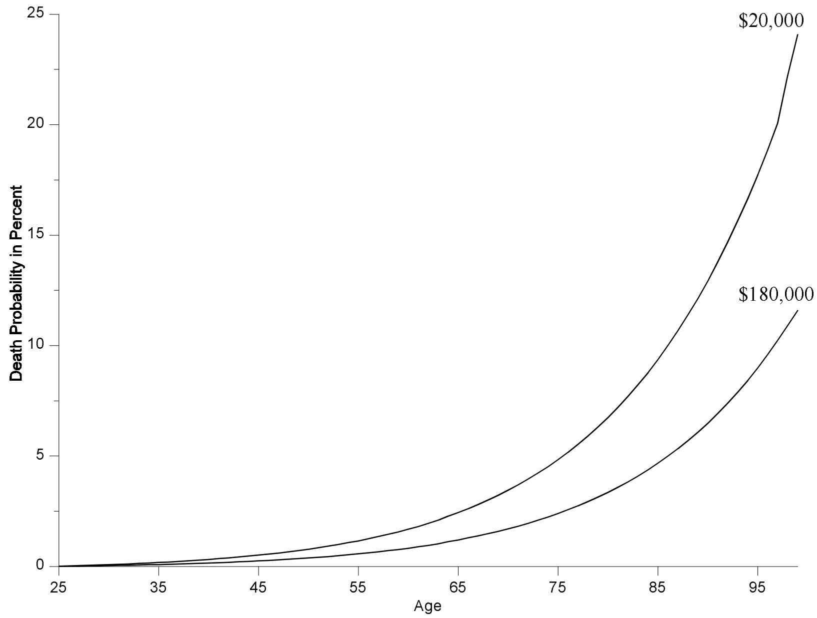


Figure 2: Load Factors for Males and Females With Social Security Benefits, r = 0.03, δ = 0.03, γ = 2.

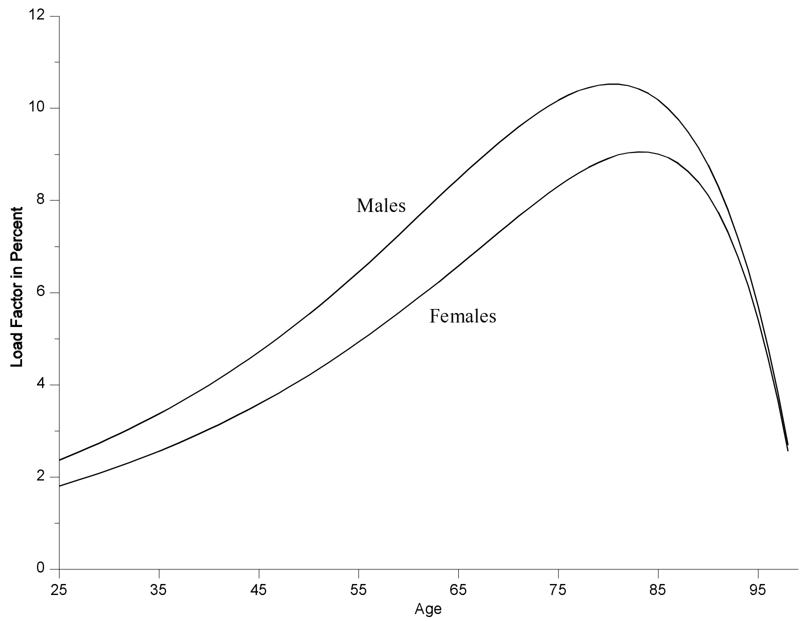


Figure 3: Load Factors for Males With and Without Social Security Benefits, r = 0.03, δ = 0.03, γ = 2.

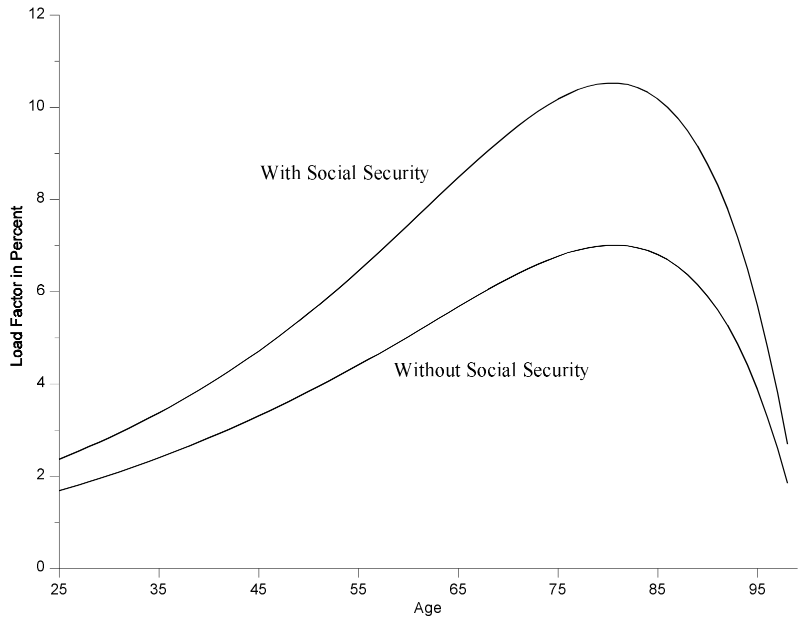


Figure 4: Consumption Paths and Private Annuity Wealth With and Without Social Security Benefits, Married Man With Income of \$10,000, r = 0.03, δ = 0.03, γ = 2.

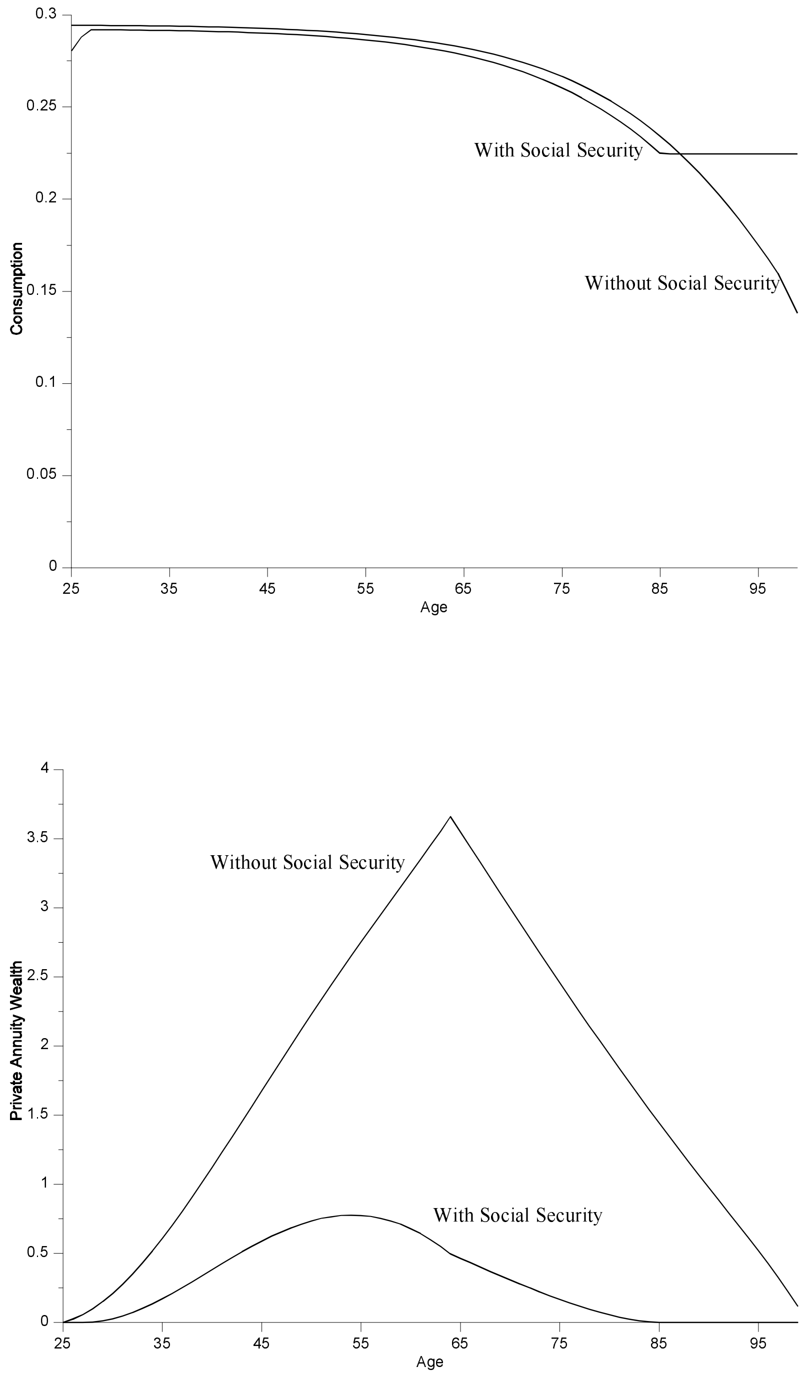
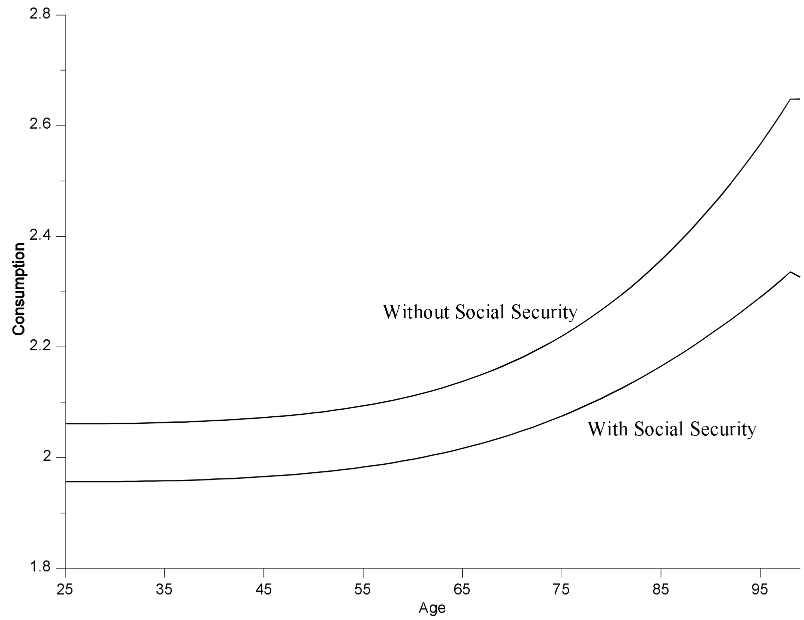
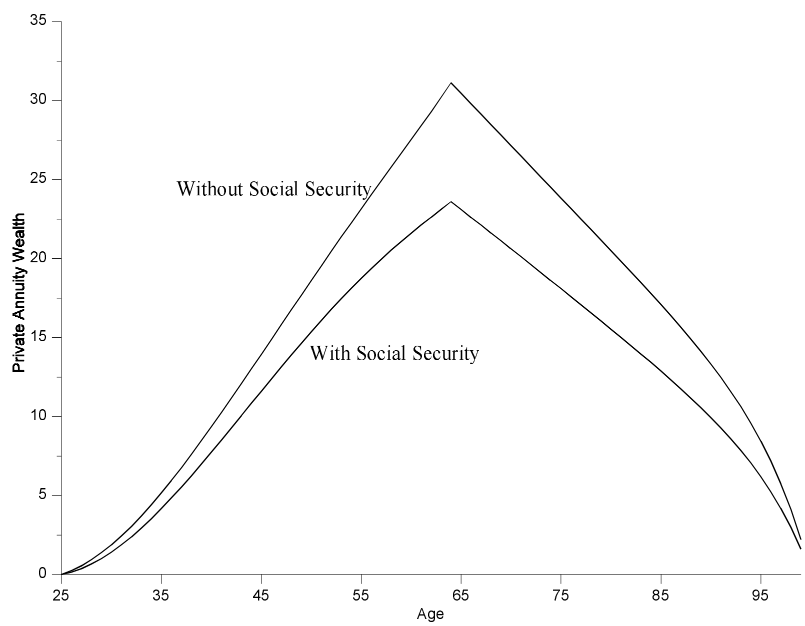


Figure 5: Consumption Paths and Private Annuity Wealth With and Without Social Security Benefits, Married Man With Income of \$90,000, $r = 0.03$, $\delta = 0.03$, $\gamma = 2$.





Appendix 1.

Proposition: A) Under assumptions A1 there exists at least one premium Z^* on the interval $(0,1/R)$ that satisfies equation (5) for an annuities market without social security or mandatory insurance. B) Under assumptions A1 and A2 there exists at least one premium Z^* on the interval $(0,1/R)$ that satisfies equation (5) if social annuities are sufficiently small.

Proof: By assumption A1 the annuity demand function a^j is continuous in Z .

A) Without the existence of social annuities, $a^j(Z) > 0$ for all $Z \in (0,1/R)$. For $Z^{min} = 0$ it holds that $(Z^{min}-\pi^j/R) < 0$ and $a^j(Z^{min}) > 0$ for all j. As a consequence, $P(Z^{min}) < 0$. Equivalently, let $1/R > Z^{max} > \pi^{max}$, where π^{max} represents the highest survival probability of any type. It follows that $(Z^{max}-\pi^j/R) > 0$ and $a^j(Z^{max}) > 0$ for all j. As a result, $P(Z^{max}) > 0$. Since $a^j(Z)$ is continuous in Z as is $(Z-\pi/R)$, expected profits $P(Z)$ are continuous in Z. Hence, on the interval $[Z^{min},Z^{max}]$ there exists at least one premium Z^* that satisfies equation (5) if annuitization is not mandatory.

B) If social security benefits are provided define $Z^{max} \in (0,1/R)$ as the premium that satisfies $a^j(Z) = 0$ for all $Z > Z^{max}$ and for all j. If social security or mandatory annuitization is sufficiently small then there exists a type k with $a^j(Z^{max}) > 0$ and $Z^{max} > \pi^k/R$. Hence, $P(Z^{max}) > 0$ and the proof to part A) applies since $P(Z)$ is continuous in Z also if social security benefits are positive.

Appendix 2.

Proposition: If private annuities markets exist, survival probabilities are not negatively correlated with income and social security benefits do not rise more than proportionally with income, the premium charged for annuities is higher than the actuarially fair premium based on average survival probabilities.

Proof: Let $\bar{\pi} = \sum_j \epsilon^j \pi^j$ and $\bar{a} = \sum_j \epsilon^j a^j$. $\bar{a} > 0$ due to the assumption that $a^j > 0$ for some j. Suppose $Z = \bar{\pi}/R$, which is the actuarial fair premium based on average survival probabilities. Therefore:

$$(5a) \qquad P(\bar{\pi}/R) = \sum_{j=1}^{N} \epsilon^j \, (\bar{\pi}/R - \pi^j/R) \; a^j(\bar{\pi}/R, \pi^j, w^j, s)$$

As a result, the sign of the profit function at $\bar{\pi}/R$ is equivalent to the sign of $-\sum_j \epsilon^j (\pi^j - \bar{\pi})\, a^j$. This expression is negative as long as $da^j/d\pi^j > 0$. Since income and survival probabilities can be correlated, $da^j/d\pi^j = (\partial a^j/\partial \pi^j) + (\partial a^j/\partial w^j)\,(\partial w^j/\partial \pi^j)$. Clearly, it follows from the first order condition in equation *(4)* that $(\partial a^j/\partial \pi^j)$ and $(\partial a^j/\partial w^j) \geq 0$ with inequality for some j under the assumption that social security benefits are not regressive. Therefore, $da^j/d\pi^j \geq 0$ with inequality for some j as long as $(\partial w^j/\partial \pi^j) \geq 0$. As a consequence, $P(\bar{\pi}/R) < 0$ if $(\partial w^j/\partial \pi^j) \geq 0$ and $Z^* > \bar{\pi}/R$. The analysis can be repeated with the same result for any $Z < \bar{\pi}/R$. Thus, in any equilibrium $1/R > Z > \bar{\pi}/R$ if $(\partial w^j/\partial \pi^j) \geq 0$.

Appendix 3.

For a two period, N type model with heterogeneity in income the annuity premium for an income proportional social annuity can be calculated analytically as long as annuity demand is positive. Otherwise an iterative strategy is needed. R^S is the internal rate of return on social security contributions.

$1/Z^* = R^A$ is the positive root of the following quadratic equation:

$$(A1) \qquad a(R^A)^2 + bR^A + c = 0$$

with:

$$(A2) \qquad a = \sum_{j=1}^{N} \frac{\epsilon^j w^j (1-s)(\pi^j)^2}{1+\delta+\pi^j}$$

$$(A3) \qquad b = -\sum_{j=1}^{N} \frac{\epsilon^j \left(Rw^j \pi^j (1-s) \; + \; \pi^j w^j s \;\; (1+\delta) R^S\right)}{1+\delta+\pi^j}$$

$$(A4) \qquad c = \sum_{j=1}^{N} \frac{R\epsilon^j w^j s \;\; (1+\delta) R^S}{(1+\delta+\pi^j)}$$